



Exploring the Characteristics of High School Students Van Hiele Thinking Levels on Quadratic Functions

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Abstract

Students' errors in understanding quadratic functions generally occur when connecting between representations. Consequently, mapping their thinking levels becomes crucial to assist teachers in optimizing instructional strategies. Therefore, this study aims to explore the characteristics of students' geometric thinking skills on quadratic functions by adapting the van Hiele model. This qualitative research involved 35 grade XI students at a high school in Jambi who had studied quadratic functions. Data were collected using a semi-structured interview guide and a 4 items test measuring geometric thinking skills from visualization to formal deduction. The results show that students' thinking skills develop from visual recognition toward abstraction of the relationships between coefficients and graph shapes, but have not reached formal deduction. At the visualization level, students classify graphs based on visual shapes. At the analysis level, they recognize the properties of quadratic functions based on coefficient values, including correlating the parabola's opening direction with minimum or maximum points. At the abstraction level, students conclude relationships between coefficient values and graph shapes. This research recommends designing quadratic function learning gradually from visual exploration toward relational abstraction before constructing formal deduction

Keywords: Geometric thinking; formal Deduction; Van Hiele model; Quadratic functions

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INTRODUCTION

Mathematics is a field of study that examines abstract objects developed through deductive reasoning processes, such as analyzing the graphs of quadratic functions. The concept of quadratic function graphs has broad applications in daily life, including project planning, economic data analysis, and technological design (Agusta, 2023). In the school curriculum, students are required not only to understand quadratic functions within their



symbolic representations but must also be capable of comprehending the relationships between these representations (Rochim et al., 2021; Rahma, 2025). Quadratic functions possess both symbolic and graphical representations, when utilized simultaneously, they can foster a deeper understanding through the process of abstraction (constructing) (*constructing*) (Mutiarani & Sofyan, 2022; Dreher et al., 2024). Strengthening conceptual understanding is a primary focus of contemporary mathematics education (Dewi et al., 2025). This learning is achieved by leveraging the visual and symbolic forms of mathematical objects, thereby facilitating the coordination of multiple representations to support conceptual understanding (Shvarts et al., 2024; Saepuloh et al., 2024). The classroom, students typically engage with graphical and symbolic algebraic representations of quadratic functions, where the instructional material is structured in a progression from visualization to formal deduction. The visual representation of quadratic functions plays a crucial role in helping students connect algebraic concepts with more concrete geometric forms. In the context of geometry learning, this framework was introduced by van Hiele (1984) as the geometric thinking model.

Geometric thinking is a vital component of mathematics, grounded in visualization, spatial reasoning, representation manipulation, and the capacity to construct and evaluate logical arguments (Shmigirilova et al., 2025). The van Hiele geometric thinking model is hierarchical, encompassing five levels: visualization, analysis, abstraction, deduction, and rigor (Sholihah & Afriansyah, 2017; Bambang, 2018). At the visualization level (Level 0), students recognize and classify quadratic function graphs based solely on their visual appearances. Moving to the analysis level (Level 1), students begin to identify specific properties of the graphs based on their coefficient values, although they cannot yet establish logical links between these properties. At the abstraction level (Level 2), students successfully deduce 'if-then' relationships between coefficients and graph behaviors, allowing them to formulate informal definitions and simple logical justifications. Advanced stages involve the deduction formal level (Level 3), where students can construct formal deductive proofs using mathematical definitions and theorems, and finally, the rigor level (Level 4), where they abstractly analyze and compare overarching mathematical systems (Vojkuvkova, 2012).

When applied to algebra, the concepts discussed in quadratic functions are not merely matters of algebraic expressions; they involve processes of visual construction and spatial relationships that are inherently geometric (Priyati & Mampouw, 2018; Giyanti & Oktaviyanthi, 2024). Converting from one representation to another indicates the extent to which students can coordinate symbolic and visual information to interpret the meaning of a function (Seet & Ishak, 2017; Murwanto et al., 2022). The ability to link symbolic and graphical representations entails identifying patterns, relationships, and function properties, alongside procedural understanding. Conversions between representations also enable students to associate changes in coefficient values with shifts in the parabola's position or shape, such as the direction of the opening and the coordinates of the vertex on the coordinate plane. This ability serves as a foundation for students to develop a profound comprehension of the structure and properties of quadratic functions, ultimately strengthening their skills in advanced mathematical reasoning and problem-solving.

In reality, a widely discussed issue in Indonesian secondary mathematics education is students' understanding of quadratic functions. Students often fail to solve problems accurately because they are confused and unable to connect symbolic representations with graphical forms (Alghadari et al., 2022; Oktaviani & Hartono, 2025). Although students may understand the impact of a specific coefficient on the shape of a graph, they frequently fail to comprehend the roles of other coefficients or the discriminant (Fauzi & Prihatnani, 2020; Syaripah, 2024). Furthermore, students still struggle to differentiate between quadratic and linear functions, and they remain unable to precisely determine intersections, the direction of the parabola, and the vertex (Khaerunnisa & Adirakasiwi, 2023). Khaerunnisa dan Adirakasiwi (2023) State that students are also unable to perform formal deductive proofs, thereby reaching only the abstraction level. A Study by Alghadari et al. (2025) reveals that students' understanding is generally confined to routine problem-solving that requires minimal deep logical reasoning. Consequently, most students experience difficulties when required to perform deductive reasoning, particularly in identifying and inferring logical relationships among the properties of quadratic functions.

Previous research has uncovered various findings regarding students' mathematical abilities in the context of quadratic functions, particularly concerning their difficulties, weaknesses, or errors. Other studies, such as Ngilamele et al. (2025) have analyzed students' mathematical representation skills in quadratic functions, while Wilkie (2024) explored the coordination across multiple representations of quadratic functions. In practice, however, many students still face significant challenges in comprehending quadratic functions, which hinders their learning progress. To overcome these difficulties, there is an urgent need to systematically map students' thinking abilities so that teachers can be better guided in designing the learning process. This study addresses this need by utilizing the van Hiele geometric thinking level as a structured diagnostic tool. Although research addressing geometric thinking ability within quadratic functions exists, it remains highly limited. Accordingly, this study aims to explore and map the characteristics of high school students' geometric thinking abilities in quadratic functions to provide concrete insights for mathematics instruction.

METHODS

This study adopts a qualitative method. It involved 35 eleventh grade students at a senior high school in Jambi City. The students were selected using a purposive sampling technique (Sugiyono, 2013). This technique was chosen to ensure that the students had already studied the material, which allowed the researcher to obtain clear and relevant information. Consequently, the sample was determined based on specific criteria: they had already studied the topic of quadratic functions and represented each level of geometric thinking according to the van Hiele model.

Research data were collected using a van Hiele geometric thinking test instrument and semi-structured interviews. The instruments were validated by two experts in mathematics education and were declared valid in terms of content, construct, and language for use in the

study. Data collection was carried out by administering a four-item written test to the students, accompanied by four multiple choice options and an open-ended column for them to write their reasoning. Each item specifically represented one level of van Hiele's geometric thinking. The first item required visualization competency to analyze several presented parabolic graphs for classification based on two specific categories. The second item required students to analyze the similarity in the representation classification of three different quadratic function examples, all of which shared a specific commonality. The third item evaluated the abstraction level, assessing whether students were able to organize multiple properties into a general relationship through the process of exploring logical connections between the coefficient components of the function and how they influence shifts in the position of the axis of symmetry and the vertex of the parabola. Lastly, the fourth item evaluated the deduction level, requiring students to reason logically by analytically evaluating the truth of a mathematical statement regarding the position of the graph's vertex based on the direction of the curve's opening and the number of its intercepts on the horizontal axis. The test instruments are limited to the formal deduction level and do not extend to rigor. This boundary is established because, according to Khaerunnisa & Adirakasiwi (2023), high school students' geometric thinking typically reaches only up to Level 3 (formal deduction), with very few achieving this stage. Consequently, this study restricts its scope strictly to the formal deduction level. Another data collection technique involved semi-structured interviews conducted with six selected subjects to understand their thinking processes and to cross-verify their answers. Those selected represented the relatively similar responses of other students. Based on the test results, no students reached the formal deduction level (Level 3). Therefore, these six subjects were chosen to represent the characteristics of the identified van Hiele levels, ranging only from visualization to abstraction. Specifically, students A1 and A2 represented the visualization level, A3 and A4 represented the analysis level, and A5 and A6 represented the abstraction level. The interviews were recorded using a smartphone voice recorder.

Once the data were collected, they were subsequently analyzed. This study utilized the Miles and Huberman data analysis model, which consists of three stages: data reduction, data display, and conclusion drawing (Miles et al., 2014). The data reduction stage involved gathering data from the test results and interviews regarding the students' thinking processes while completing the test. The next stage was data display, wherein the obtained data were presented by describing the students answers and interviews using text and tables. In the final stage, conclusion drawing, the researchers explained the data results to provide an overview of the van Hiele geometric thinking levels in quadratic functions.

RESULTS AND DISCUSSION

Based on the research findings, the data obtained from students' test responses and interviews regarding geometric thinking abilities in the concept of quadratic functions adapted from the van Hiele model are analyzed and summarized as follows.

Table 1. Data summary of test results

Student Code	Item	Description of reasoning for answers to the item
A1	Visualization	Classification of graphs based on whether they open upward or downward, and whether the graph intersects or does not intersect the x axis, as visible in the presented images.
A2	Visualization	Classification of graphs based on whether the coefficient of x^2 is positive or negative.
A3	Analysis	The common property of the graphs for the three presented functions is that their vertices are located above the x -axis due to the positive coefficient of x^2 .
A4	Analysis	The common property of the graphs for the three presented functions is that they have a minimum value because the constant value is positive.
A5	Analysis	The common property of the graphs for the three presented functions is that they have a minimum value because the functions are classified as opening upward.
A1	Abstraction	The relationship between the signs and values of coefficients a and b and the vertex position of the function $f(x)=ax^2+bx+c$, $a > 0$ is that the larger a is, the further to the right the vertex position always shifts because a is always positive.
A6	Abstraction	The relationship between the signs and values of coefficients a and b and the vertex position of the function $f(x)=ax^2+bx+c$, $a > 0$ is that the value of b affects the location of the parabola's axis of symmetry because the axis of symmetry is $x = (-b)/2a$, sehingga nilai b menentukan posisi horizontal dari puncak parabola.
A4	Deduction	The truth of the statement "if the graph of a quadratic function has two intercepts with the x -axis and opens upward, then its vertex is below the x -axis cannot be determined without drawing the graph. The graph could face downward while the equation is unknown, or the vertex at x does not intersect.
A5	Deduction	The statement "If the graph of a quadratic function has two intercepts with the x -axis and opens upward, then its vertex is below the x -axis" is true based on $D > 0$ and a negative minimum value.
A6	Deduction	The statement "If the graph of a quadratic function has two intercepts with the x -axis and opens upward, then its vertex is below the x -axis" is true with $D > 0$ and a negative minimum value because $D > 0$ and $a > 0$, so the graph opens upward.

Regarding the visualization test results, an examination of all responses revealed that all students ($N=35$) were able to answer the visualization test item, albeit with varying reasons. As shown in Table 1, Student A1 explained that the distinction in graph classification was based on the direction of their opening and whether or not they intersected the x -axis. Data from the interview results in Table 2 confirmed that A1 answered the visualization item based purely on visual appearance. Furthermore, Student A2 explained that the graph classification differed based on the value of coefficient a , identifying a group of graphs with a positive a value and another with a negative a value, in addition to the direction of the parabola's opening.

Table 2. Data summary of interview results

Student siswa	Item	Description of confirmation of answers or reasoning
A1	Visualization	Classification of shapes corresponds to the visualization of the presented images.
A2	Visualization	Another classification of shapes corresponds to the direction of the parabola opening upward or downward.
A4	Analysis	There are no other common properties.
A5	Analysis	Because $a > 0$, the function opens upward, so the common property is that they all have a minimum value.
A1	Analysis	Perhaps because the value of b determines its value.
A6	Abstraction	The formula for the axis of symmetry line is $x = (-b)/2a$, and the position of the line is determined by b , for instance, if the value of b is positive, the line shifts toward the positive x -axis.
A4	Abstraction	Does not understand the meaning of "intersecting the x -axis at two distinct points."
A5	Deduction	If a graph image is provided, the question can be answered.
A6	Deduction	If $D = 0$, then the intercepts are identical.

For the test results from the analysis level item, the findings reveal that students identified the properties of quadratic function graphs inductively; however, their logical justifications remained inconsistent, and some were mistaken in linking coefficients to graph properties. Several students were able to answer correctly with different reasons, while others answered incorrectly. To verify and elaborate on these test results shown in Table 1, the interview analysis highlights the most distinct and representative responses among the subjects, avoiding redundancy since some shared nearly identical reasoning. For instance, Student A5 stated that the commonality among functions within the same classification group included having a minimum or maximum value and a specific opening direction. Furthermore, Student A5 already knew that these features constitute part of the properties of quadratic functions, which was confirmed by their statement during the interview. In the same context, Student A4 provided an inaccurate rationale, despite stating that having a minimum value was the sole common property for the three function models presented in the item. Meanwhile, Student A3 provided an inaccurate answer even though they knew that all three functions shared a positive a value. Student A3 did not demonstrate an understanding that a positive a value means the parabola will open upward or possess a minimum value.

On the abstraction item, the findings of this study reveal that students recognized the role of a coefficient in determining the horizontal position of the parabola's vertex via the axis of symmetry formula, yet they did not accurately comprehend the specific influence of certain coefficients. In accordance with Table 1 and Table 2, some students answered correctly while others answered incorrectly. As in the response from Student A5, they knew the relationship between coefficients a and b in determining the horizontal position toward the positive or negative x -axis, which was based on the axis of symmetry calculation formula. Conversely, several other students provided inaccurate answers; for example, Student A1 stated that the larger the value of a , the further to the right the vertex position shifts. In reality, coefficient a

only affects the direction of the opening rather than the location of the axis of symmetry, and Student A1 did not yet know the influence of coefficient b on the quadratic function graph as the determinant of the axis of symmetry position.

For the deduction level item, students did not yet demonstrate a formal deductive process or verify the truth of statements based on a systematic relationship among properties. Consequently, there is no indication that any specific student was able to attain the deduction level. One student (A4) did not demonstrate an understanding of the function's discriminant value, instead indicating a reliance on graphic illustrations. Although some students (A5 and A7) were able to answer correctly, none of them provided reasoning based on a coherent sequence of relationships between the properties of quadratic functions that would be deductively appropriate for drawing a conclusion.

Characteristics of Students Geometric Thinking

At the visualization level, the primary characteristic is that students classify quadratic function graphs entirely based on visual attributes. Essentially, this characteristic relies on three main aspects: grouping quadratic function graphs based on the shape and direction of the parabola's opening, identifying visual similarities between graphs, and a strong tendency among students to prioritize visualization (Muhassanah et al., 2025). This aligns with the assertion by Fauzi and Prihatnani (2020) that students still rely heavily on visualization to recognize quadratic functions as parabolas opening either upward or downward. Within the context of quadratic function graphs, students' geometric thinking should ideally progress toward a transitional phase that analyzes functional properties in greater depth (Hutajulu et al., 2022; Nursyahidah et al., 2025). Nevertheless, this cognitive development process often encounters significant obstacles, where students become trapped in an over-reliance on purely visual representations, thereby hindering their advancement to higher levels. This finding is consistent with Alghadari et al. (2022) which noted that students failed to solve problems accurately because they were confused and unable to link symbolic representations with graphical representations.

At the analysis level, the characteristic of students' geometric thinking in constructing quadratic functions is generally marked by the ability to understand function properties based on coefficient values, including correlating the direction of the parabola's opening with the existence of a minimum point. Although the majority of students operate within this analysis phase, their cognitive processes are not free from errors. These errors often manifest as conceptual misconceptions regarding the properties of quadratic functions and mistakes in mapping the relationships between their constituent elements onto the graph (Mataheru et al., 2021; Nalim et al., 2026). This condition is evident when some students remain unable to precisely determine intercepts, the direction of the parabola, and the vertex (Khaerunnisa & Adirakasiwi, 2023). Additionally, several students encounter difficulties when connecting coefficients to the minimum or maximum points (Alghadari et al., 2019). To bridge these challenges, the use of graphical representations plays a crucial role in geometric thinking, as it

not only eases the visualization of abstract objects but also stimulates a more comprehensive analytical capacity in students.

At the abstraction level, the characteristic of students' geometric thinking is characterized by the ability to organize multiple properties into a general relationship (Wibawa et al., 2024; Hutajulu et al., 2022; Usman, 2017) such as understanding the general relationships between coefficients and the behavior of quadratic function graphs without depending on visual aids. This corresponds to research by Fauzi and Prihatnani (2020) which indicated that students understand the influence of coefficients on the shape of the graph. During the transition to this abstract reasoning phase, students frequently encounter cognitive barriers in the form of fundamental misconceptions. For example, they may conflate the roles of different coefficients when determining the position of the vertex or the opening direction of the parabola, which triggers hesitation and inconsistent arguments (Alghadari et al., 2019). For certain students, the axis of symmetry formula serves as a highly crucial logical tool to support arguments regarding the direction of the graph's shift.

Students' failure to fully reach the formal deduction level is reflected in their inability to construct arguments within a coherent framework of logical reasoning. Instead of formulating deductive arguments to verify the validity of a statement, students often fall into circular reasoning by merely restating the components of the problem. This limitation is partly influenced by a reliance on visual images, where students face challenges when required to abstract the properties of quadratic functions without the aid of graphical stimuli (Mukarromah et al., 2024). Consequently, their formal generalizations become biased, triggering fundamental systemic misconceptions when mapping logical relationships among function elements. A prominent example is the fatal error of swapping the functional roles of the discriminant value (D) and the coefficient a when predicting the position and orientation of the graph on the coordinate system. This finding aligns with Alghadari et al. (2025) which revealed that students' understanding is generally confined to routine problem-solving that demands minimal deep logical reasoning. As a result, most students face difficulties when executing deductive reasoning, particularly in identifying and inferring logical relationships among the properties of quadratic functions. This is further supported by research from Khaerunnisa and Adirakasiwi (2023) which states that students are not yet capable of performing formal deductive proofs, thereby limiting them to the abstraction level.

Based on the results of this study, the van Hiele thinking model can also be applied to quadratic functions, rather than being restricted solely to conventional geometry topics. This is due to the distinct characteristics observed at each thinking level within the topic of quadratic functions, ranging from the visualization level to the formal deduction level. Furthermore, the findings of this study reveal that quadratic function instruction needs to strengthen students' abilities at lower levels before introducing concepts at subsequent levels. For instance, prior to engaging in learning activities relevant to the formal deduction level, the instructional design must ensure that students have already reached the abstraction level. Instructional designs can also leverage software such as GeoGebra to address students' dependence on visual images.

Therefore, this study recommends that quadratic function learning be designed progressively, starting from visual exploration and moving toward relational abstraction before constructing formal deduction.

CONCLUSION

The characteristics of students' geometric thinking abilities in quadratic functions develop progressively but have not yet reached the formal deduction level. Students are capable of classifying graphs based on visible visual attributes, marking achievement at the visualization level; however, their inductive identification of graph properties is not yet accompanied by consistent logical justifications, leaving the connection between symbolic and graphical representations weak. Although students recognize the role of a specific coefficient in quadratic functions, their understanding of the roles of other coefficients remains erroneous and partial, indicating that relational abstraction among the properties of quadratic functions has not yet fully formed. Furthermore, students are entirely unable to perform formal deduction to verify the truth of statements based on a systematic relationship between properties. Consequently, students' geometric thinking abilities remain confined to the boundary between the analysis and abstraction levels within the van Hiele framework, unable to advance toward rigorous deductive reasoning. The weak connection between symbolic and graphical representations, coupled with the absence of formal deduction, indicates that conventional quadratic function instruction has not yet constructed students' geometric thinking structures hierarchically and meaningfully. Accordingly, quadratic function instruction needs to be explicitly designed based on the van Hiele stages, progressing from visual exploration to relational abstraction, before requiring students to engage in formal deductive reasoning.

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