SJME (Supremum Journal of Mathematics Education)
Vol.10, No.2, July 2026, pp. 245-261
ISSN: 2548-8163 (online)| ISSN: 2549-3639 (print)
DOI: 10.35706/sjme. v10i2.13478

Hypothetical Learning Trajectory for Seventh Graders' Understanding of Squares through Techno-Ethno-Realistic Mathematics Education

Farida Nursyahidah[1*], Maya Rini Rubowo[2], Arif Wibisono[3], Khoiriya Latifah[4], Windia Hadi[5]
[1*,2,3,4] Universitas PGRI Semarang, Semarang, Indonesia
[1,3] PUI-PT Techno-Ethno-Realistic Mathematics Education, Universitas PGRI Semarang, Semarang, Indonesia
[5] Doctoral School of Education, University of Szeged, Hungary
**Corresponding author*
Email: faridanursyahidah@upgris.ac.id[1*], mayarinirubowo@upgris.ac.id[2], arifwibisono@upgris.ac.id[3], khoiriyalatifah@upgris.ac.id[4], windia.hadi@edu.u-szeged.hu[5]

Article Information
Received May 31st, 2026
Revised June 28th, 2026
Accepted July 02nd, 2026

Diterima 31 Mei 2026
Direvisi 26 Juni 2026
Disetujui 02 Juli 2026

Keywords:
Ethnomathematics; GeoGebra; Realistic Mathematics Education; Technology; Quadrilaterals

Kata kunci: Etnomatematika; GeoGebra; Pendidikan Matematika Realistik; Teknologi; Segiempat

ABSTRACT
The concept of squares is a fundamental topic in junior high school geometry because it serves as a foundation for understanding quadrilaterals and more advanced geometric concepts. However, many seventh-grade students have difficulty distinguishing the properties of squares from those of other quadrilaterals and often rely on procedural understanding rather than conceptual reasoning. To address this challenge, this study developed a Hypothetical Learning Trajectory (HLT) to support seventh-grade students' understanding of squares through Techno-Ethno-Realistic Mathematics Education (TE-RME). This study employed a design research methodology consisting of three stages: preliminary design, teaching experiments, and retrospective analysis. The participants were seventh-grade students at a junior high school in Semarang. This article focuses on the preliminary design stage, namely the development of an HLT comprising three main activities: (1) observing a contextual video; (2) exploring the perimeter of a square using sticks and GeoGebra; and (3) exploring the area of a square using sticks, origami paper, and GeoGebra. These activities were designed to help students build a deeper understanding of square concepts while fostering appreciation of local culture and diversity in mathematical thinking. The proposed HLT is intended for implementation in experimental classrooms and research-based teaching using TE-RME.

ABSTRAK
Konsep persegi merupakan topik fundamental dalam geometri SMP karena menjadi dasar untuk memahami segiempat dan konsep geometri yang lebih lanjut. Namun, banyak siswa kelas tujuh masih mengalami kesulitan dalam membedakan sifat-sifat persegi dari segiempat lainnya dan cenderung mengandalkan pemahaman prosedural daripada penalaran konseptual. Untuk mengatasi tantangan tersebut, penelitian ini mengembangkan Hypothetical Learning Trajectory (HLT) untuk mendukung pemahaman siswa kelas tujuh tentang persegi melalui pendekatan Techno-Ethno-Realistic Mathematics Education (TE-RME). Penelitian ini menggunakan metode design research yang terdiri atas tiga tahap: desain awal, eksperimen pengajaran, dan analisis retrospektif. Subjek penelitian adalah siswa kelas tujuh di salah satu SMP di Semarang. Artikel ini berfokus pada tahap desain awal, yaitu pengembangan HLT yang

mencakup tiga aktivitas utama: (1) mengamati video kontekstual; (2) mengeksplorasi keliling persegi menggunakan lidi dan GeoGebra; dan (3) mengeksplorasi luas persegi menggunakan lidi, kertas origami, dan GeoGebra. Rangkaian aktivitas ini dirancang untuk membantu siswa membangun pemahaman yang lebih mendalam tentang konsep persegi sekaligus menumbuhkan apresiasi terhadap budaya lokal dan keragaman dalam berpikir matematis. HLT yang diusulkan dirancang untuk diimplementasikan dalam kelas eksperimen dan pembelajaran berbasis penelitian menggunakan TE-RME.

INTRODUCTION

Geometry is a fundamental domain of mathematics because it plays an important role in developing students' spatial reasoning, visualization, logical thinking, and problem-solving abilities (Clements & Battista, 1992). Through geometry learning, students are expected to understand shapes, spatial relationships, and mathematical structures that are closely connected to everyday life (NCTM, 2000). One of the essential topics in junior high school geometry is the concept of a square. Understanding squares is not only important for learning perimeter and area but also serves as a foundation for understanding quadrilaterals, similarity, congruence, transformations, and other advanced geometric concepts (Fujita & Jones, 2007). Previous studies have emphasized that students' geometric understanding contributes significantly to the development of higher-order mathematical thinking and supports success in subsequent mathematics learning (Yuliardi & Rosjanuardi, 2021). Furthermore, geometry learning enables students to connect abstract mathematical ideas with real-world situations through visual and spatial representations, making it a crucial component of meaningful mathematics education. Therefore, developing students' conceptual understanding of squares is an important objective in mathematics learning.

Despite its importance, numerous studies have reported that students continue to experience difficulties in learning quadrilateral concepts, particularly squares. In the classification domain, students often rely heavily on prototypical images. As a result, they may fail to recognize that a square is a special type of rectangle and may also misidentify rotated shapes, such as considering a square rotated 45° to be a rhombus. This barrier is consistent with the findings of Fujita and Jones (2007), who examined students' difficulties in assimilating formal definitions into their conceptual images. These difficulties also appear in geometric calculations, particularly in the concepts of perimeter and area, where students tend to memorize formulas mechanically without developing spatial understanding. For perimeter, students often confuse the boundary of a shape with its area, especially when working with compound figures; they tend to add all numbers shown in a figure, including inner sides that are not part of the outer boundary. For area, students often experience structural misconceptions and mistakenly believe that two shapes with the same perimeter must also have the same area. These misconceptions occur because instruction often skips crucial conceptual stages, such as constructing unit squares or physically manipulating objects, and proceeds directly to formal formulas. At a more abstract level, students also face significant challenges in understanding class inclusion. According to van Hiele's developmental theory, many students remain at the analytical level and have not yet progressed to the informal deduction level.

Many students are unable to distinguish the defining properties of squares from those of other quadrilaterals and tend to rely on memorized formulas rather than conceptual reasoning. Students frequently perceive squares and rectangles as separate and unrelated figures, which leads to misconceptions about the hierarchical relationships among quadrilaterals. Research on quadrilateral learning has shown that students often encounter obstacles in identifying attributes, analyzing relationships among shapes, and constructing mathematical arguments based on geometric properties (Amelia & Herman, 2025). Similar findings indicate that weaknesses in spatial conception and geometric reasoning significantly influence students' understanding of geometric concepts, including squares and other quadrilaterals (Yuliardi & Rosjanuardi, 2021). These findings suggest a substantial gap between expected learning outcomes and students' actual conceptual understanding of squares.

The difficulties students experience are influenced by several factors. One important factor is the tendency of classroom instruction to focus on procedural activities and formula application rather than on supporting students' conceptual construction processes (Angraini et al., 2023). In many cases, teachers have not yet optimally designed learning trajectories that anticipate students' thinking processes and learning obstacles (Salsabila & Hajizah, 2023). As a result, learning activities may not facilitate progressive conceptual development. In addition, the integration of technology in geometry learning remains limited, even though technological tools can help students visualize geometric relationships dynamically (Zakaria et al., 2024). Research has demonstrated that many students struggle with abstract geometric concepts because learning resources do not sufficiently support visual exploration and conceptual understanding. Another contributing factor is the lack of meaningful contexts that connect mathematical concepts to students' cultural and everyday experiences (Lipták, 2025). Consequently, students frequently perceive mathematics as detached from real life and difficult to understand. Studies on ethnomathematics have highlighted that geometry learning often overlooks local cultural contexts that could serve as meaningful starting points for mathematical exploration (Nursyahidah et al., 2025; Nursyahidah et al., 2025c).

To address these challenges, innovative learning designs are needed to facilitate students' meaningful understanding of square concepts. One promising approach is the integration of technology, ethnomathematics, and Realistic Mathematics Education (RME), structurally synthesized into a coherent instructional framework known as Techno-Ethno-Realistic Mathematics Education (TE-RME) (Nursyahidah et al., 2025a, 2025b). Within this unified framework, these three components do not operate in isolation; instead, they function simultaneously to drive students' progressive mathematization processes.

This integration is organized as a planned learning flow: ethnomathematics acts as an initial foundation that provides meaning, RME guides the learning process, and technology broadens and sharpens students' mathematical understanding. First, ethnomathematics enables students to explore mathematical ideas embedded in their cultural environment, such as the architectural symmetries of local heritage, making mathematics more relevant to their lives (Prahmana & D'Ambrosio, 2020). Second, this cultural context is operationalized using RME principles, which treat the heritage site not merely as a passive backdrop but as a realistic situation that triggers guided reinvention through progressive modeling and collaborative interaction (Nursyahidah et al., 2025c). Third, technology is infused into this RME-guided cultural exploration to overcome the limitations of static physical materials. By using contextual videos and dynamic GeoGebra activities, technology provides interactive visualizations and real-time feedback (Suparman et al., 2024).

Together, these components form a closed loop: ethnomathematics offers a culturally responsive problem-solving scenario, RME structures the social and conceptual trajectory from informal to formal mathematics, and GeoGebra enables students to dynamically investigate and generalize the geometric properties, perimeter, and area formulas discovered within the cultural artifact. Previous studies support this synergy, demonstrating that combining digital tools such as GeoGebra with RME significantly improves students' conceptual understanding of geometry (Ramadhan et al., 2025), while connecting instruction with local wisdom enhances engagement, cultural awareness, and mathematical reasoning (Nursyahidah & Albab, 2021).

Although various studies have developed learning designs for geometry using ethnomathematical contexts such as temples (Sulistyowati & Khotimah, 2022), traditional houses (Nurtriana & Nuryadi, 2025), cak ingkling traditional games (Disnawati et al., 2015), and batik motifs (Amaliyah & Marsigit, 2024; Prahmana & D'Ambrosio, 2020), research focusing on square learning through the context of Lawang Sewu remains very limited. Therefore, this study offers a novel contribution by systematically using the historical architecture of Lawang Sewu as an ethnomathematical foundation for geometry instruction. Lawang Sewu is mathematically appropriate for learning squares because of the geometric regularity and symmetry embedded in its Dutch colonial design. Specifically, the primary architectural elements that represent square properties can be found in its distinctive arched window frames and structural door panels. These components exhibit four congruent sides and orthogonal intersections that form precise 90° right angles, providing students with concrete, real-world examples of a square's critical attributes that remain invariant regardless of visual perspective.

Furthermore, the prominent concrete pillars and linear corridors of the building inherently map onto the concept of perimeter. The continuous outer masonry borders that frame these structural sections create a visible physical boundary, allowing students to conceptualize perimeter as the total linear distance of an enclosed path (4s). In addition, the highly uniform grid patterns of the historic floor tiles inside the main halls serve as an ideal spatial model for exploring area. This regular tile tessellation explicitly demonstrates the concept of area as the coverage of a continuous space. It provides a structured row-and-column array that supports students' cognitive shift from counting individual unit squares toward abstract multiplicative reasoning (side × side). By mapping these specific architectural attributes to target geometric concepts, Lawang Sewu serves as a theoretically robust vehicle for progressive mathematization, supported by contextual videos and GeoGebra.

The use of video contexts enables students to observe authentic cultural objects and connect geometric concepts with real-world situations more concretely (Hidayat & Firmanti, 2024). Video-based learning also supports students' attention, motivation, and contextual understanding by presenting mathematical phenomena in a meaningful visual format (Nursyahidah et al., 2023; Noetel et al., 2021). Furthermore, GeoGebra provides dynamic representations that allow students to manipulate geometric objects, investigate properties of squares, and construct mathematical relationships interactively (Suparman et al., 2024). Previous research has shown that GeoGebra-based geometry learning significantly enhances conceptual understanding and supports students in exploring geometric structures more effectively (Larico et al., 2026).

Based on the aforementioned issues, developing a carefully designed learning trajectory is essential to support students' conceptual understanding of squares. Within the framework of design research, a Hypothetical Learning Trajectory (HLT) functions as an instructional hypothesis that predicts students' learning processes, possible strategies, and potential learning obstacles during instruction. An HLT enables researchers and teachers to design learning activities that align with students' cognitive development while facilitating the transition from informal understanding to formal mathematical reasoning. Therefore, the purpose of this study is to design and formulate a Hypothetical Learning Trajectory for seventh graders' understanding of squares through Techno-Ethno-Realistic Mathematics Education (TE-RME) using the Lawang Sewu context, supported by contextual videos and GeoGebra. Reflecting the scope of the preliminary design phase, this paper focuses on the theoretical development and diagnostic configuration of the trajectory. The resulting hypothetical learning trajectory is expected to contribute theoretically to the foundational advancement of design research in mathematics education and practically by providing a conceptual framework for implementing culturally responsive and technology-enhanced geometry learning.

METHOD

Employing design research as a methodological framework for developing educational tools and learning theories (Bakker, 2018), this study was structured into three distinct phases following the Gravemeijer and Cobb (2006) model: (1) preliminary design, (2) design experiments, and (3) retrospective analysis. However, this paper reports exclusively on the findings and theoretical development of the preliminary design phase.

During this initial phase, a selected group of 32 seventh graders from a public junior high school in Semarang contributed as a diagnostic cohort rather than as participants in a full-scale teaching experiment. Their role was to inform the baseline configuration of the instructional design. To thoroughly map students' baseline competencies and learning requirements for mastering the properties of a square, a targeted preliminary assessment was administered. This baseline assessment was analyzed specifically to identify prevalent geometric misconceptions, visual biases, and existing informal knowledge related to quadrilaterals within the student population.

Consequently, the data sources reported in this phase were strictly calibrated to support the formulation of the Hypothetical Learning Trajectory (HLT). Rather than analyzing live classroom interactions, the data used in this preliminary stage were drawn from: (1) diagnostic pre-assessments to capture students' initial problem-solving capacities and conceptual readiness; and (2) a critical evaluation of the structured student worksheets and instructional materials during their initial drafting. Other data sources, such as live classroom video footage, observation logs, and post-assessments, were established as part of the overarching research design for the subsequent design experiment phase and are therefore outside the analytical scope of this paper.

Based on the diagnosed learning requirements, the HLT was formulated by integrating the local cultural context of Lawang Sewu. To support this theoretical trajectory, a contextual video and GeoGebra activities were developed as concrete pedagogical tools designed to visually introduce the ethnomathematical context and demonstrate how specific heritage elements mathematically correspond to the properties of a square.

RESULTS AND DISCUSSION

During the preliminary phase of this design research, several key activities were carried out, including a literature review, an assessment of students' prior knowledge, and the development of a Hypothetical Learning Trajectory (HLT). The literature review involved gathering foundational materials on the geometry of squares, the TE-RME framework, and design research methodologies to establish an initial instructional strategy.

Following this, students' baseline abilities were assessed to determine their prerequisite knowledge. These preparatory steps culminated in the creation of a dynamic HLT, intentionally designed to be flexible and adjustable during the teaching experiments (Bakker, 2018). Accordingly, this section details the evaluation of students' competencies and the formulation of the HLT.

To effectively grasp seventh-grade quadrilateral concepts, students must build upon a set of foundational competencies acquired during their elementary years. The progression of these core skills, spanning from first to sixth grade, is structured as follows. Phases A and B (Grades 1 to 3) focus on introducing basic plane figures and spatial awareness. First graders are expected to recognize and classify simple two-dimensional shapes, such as circles, triangles, and quadrilaterals, by observing concrete objects in their surroundings. In second grade, the curriculum introduces the concept of angles formed by intersecting lines, requiring students to identify right, acute, and obtuse angles. By third grade, learners begin to recognize the specific properties of various plane figures and learn to calculate perimeters by summing the lengths of all outer sides.

Moving into Phase C (Grades 4 to 6), the focus shifts toward more advanced geometry and measurement. Fourth graders learn to identify distinct types of quadrilaterals, including squares, rectangles, parallelograms, and trapezoids, based on their specific geometric characteristics. They also explore the concepts of line and rotational symmetry. In fifth grade, students apply standard formulas to compute the area of various quadrilaterals and learn to deduce side lengths when given the total area or perimeter. Finally, sixth-grade mathematics introduces the Cartesian coordinate system; understanding point locations at this stage serves as a crucial stepping stone for graphing quadrilaterals on a coordinate plane in subsequent levels.

Mastering these foundational skills significantly eases students' transition into seventh-grade geometry. At this junior high school level, the learning focus shifts toward a more investigative approach, where students are guided to reinvent the perimeter and area formulas for squares, rectangles, parallelograms, trapezoids, rhombuses, and kites, while deeply analyzing the lines and angles that construct these geometric shapes.

Following the literature review and the assessment of students' foundational and required competencies, the subsequent phase involved designing a sequence of instructional activities centered on the concept of squares. These activities were formulated into a Hypothetical Learning Trajectory (HLT) anchored in the local cultural context of Lawang Sewu in Semarang, Central Java. The proposed HLT outlines three distinct instructional procedures. Supported by contextual video media, these steps were designed not only to facilitate conceptual understanding but also to inspire students and foster an engaging, meaningful learning experience. The three activities are detailed below.

The primary objective of this initial stage is to enable students to identify square shapes through a virtual exploration of the Lawang Sewu heritage building using a contextual video. The phase begins with students watching the video and then completing a worksheet based on their visual findings. Rather than assuming a linear path of successful identification, the HLT for this activity anticipates critical student errors and informal reasoning strategies.

First, the design predicts that students may initially identify shapes based purely on their holistic visual appearance, making them susceptible to orientation biases in which a square rotated at an angle is misidentified as a non-square rhombus. Furthermore, students are expected to struggle with class inclusion, frequently confusing squares with general rectangles or treating them as completely unrelated categories.

To address these anticipated difficulties, the worksheet is systematically structured to scaffold students' cognitive processes. It moves students beyond passive observation by presenting contrasting prompt questions that require them to analyze non-prototypical geometric properties (for example, "If we change the viewing angle of this window, do its side lengths and right angles change?"). Throughout this exercise, the teacher serves as a facilitator, guiding learners to resolve visual misconceptions and explore the cultural context more deeply while offering supplementary insights when necessary. Figure 1 illustrates the culturally rich architectural details of Lawang Sewu that students observe during this instructional session.

Figure 1. Structural symmetries and orthogonal window frames of the *Lawang Sewu* cultural heritage architecture, serving as a visual context for identifying critical attributes of a square

As depicted in Figure 1, the visual material highlights the explicit geometric features embedded within the Lawang Sewu architecture, specifically the four equal side lengths and the perpendicular 90° intersections of its prominent window and door frames. In Activity 1, students use these distinct visual prompts to develop their geometric understanding by transitioning from holistic visual perception to analytic, property-based reasoning. By verifying that these architectural elements retain their congruent sides and right angles despite varying photographic perspectives, students are guided to abstract the defining properties of a square and avoid the common misconception of identifying shapes based solely on prototypical orientations. Students' assumed thinking in Activity 1 is shown in Table 1.

Table 1. Students' Assumed Thinking in Activity 1

No	Activity	Expected Student Thinking & Informal Strategies	Potential Errors & Misconceptions	Teacher's Scaffolding / Intervention
1	Engage with the context-based video about *Lawang Sewu*	Visual-Holistic Notice: Students identify shapes based on their overall physical look (for example, matching shapes to familiar objects like doors or windows).	Orientation Bias: Students only recognize a square when its base is perfectly horizontal. They fail to identify a square captured from a dynamic/skewed angle in the video,	Present a freeze-frame of the skewed shape and ask: *"If we physically step to the front of this window, will the side lengths or the corners change their shapes?"*

No	Activity	Expected Student Thinking & Informal Strategies	Potential Errors & Misconceptions	Teacher's Scaffolding / Intervention
		Property-Based Observation: Students informally point out identical side lengths or straight/upright corners in the architecture.	mistaking it for a generic rhombus. Over-generalization: Students visually clump all four-sided architectural features together without checking properties.	Encourage students to trace the boundaries digitally or on paper to focus on the lines rather than the background orientation.
2	Perform Activity 1	Descriptive Analysis: Students use informal language to define properties (for example, all four sides are equal and the corners form perfect L-shapes).	Visual vs. Conceptual Conflict: Students identify a shape by name but fail to list its critical attributes correctly.	To address class inclusion: Provide a guiding question: *"What is the minimum rule for a shape to be called a rectangle? Does a square meet all of those requirements?"*
		Class Comparison: Students contrast squares with rectangles by focusing on the length of adjacent sides.	Lack of Class Inclusion: Students treat squares and rectangles as entirely separate, mutually exclusive groups, rigidly claiming that a square cannot be a rectangle.	

As reconstructed in Table 1, the HLT for Activity 1 shifts away from linear, outcome-oriented predictions toward a multi-layered conjecture of student reasoning. The design explicitly maps the cognitive transition from visual-holistic noticing to descriptive property analysis. By anticipating specific geometric hurdles, such as orientation biases from the video perspective and structural classification errors regarding class inclusion, the framework incorporates immediate and responsive scaffolding prompts. This ensures that the instructor can turn predictable student errors into productive mathematical turning points during instruction. Reflecting on Activity 1, students are expected to achieve the learning objectives by identifying various quadrilaterals and deducing their properties. For students who encounter difficulties during this process, the teacher provides targeted scaffolding. The teacher's role is to guide students in organizing their findings and ultimately help them construct a solid conceptual understanding of quadrilaterals, with a particular focus on squares.

The primary learning objective of this second phase is to guide students in determining the perimeter of a square. The session begins with students observing the contextual video of Lawang Sewu, which guides them to describe the physical attributes of a square based on its architectural symmetry. Subsequently, students model the perimeter using physical sticks of equal length to represent the four outer boundaries. To facilitate the transition from this concrete model to mathematical generalization, instruction moves through progressive abstraction. First, as students arrange four identical sticks to enclose the square, they use an informal additive strategy, recording the total length as side + side + side + side. The teacher then introduces a cognitive conflict by asking students to predict the total boundary if the square size increases significantly, encouraging them to seek a more efficient shortcut. This prompts students to recognize the algebraic structure of repeated addition with identical lengths.

Once students grasp this conceptual repetition through the tangible model, they transition to digital exploration using GeoGebra. In GeoGebra, students manipulate dynamic sliders to change the side length (s) and observe how the total boundary length changes dynamically in a 1:4 ratio. By connecting the physical action of assembling four identical sticks to the digital covariation in GeoGebra, students formalize their informal additive thinking into the abstract multiplicative formula for the perimeter of a square: $P = 4s$. Figure 2 illustrates the square patterns found within the architecture of Lawang Sewu that initiate this trajectory.

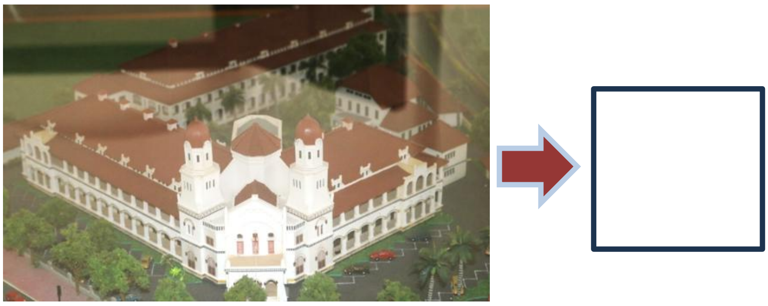

Figure 2. The architecture of *Lawang Sewu* serves as a contextual basis for learning the perimeter of a square

In a practical exercise, students are given sticks and challenged to arrange them into a square. They are then asked to sketch the shape, a relatively straightforward step for learners. They proceed to explore the perimeter dynamically in GeoGebra. After successfully determining the perimeter, the activity deliberately guides students to apply this knowledge to solve practical, contextual problems.

Through this progressive sequence, students are expected to independently deduce the perimeter formula and apply it effectively in problem solving. When learners struggle with these tasks, the teacher acts as a facilitator by providing the necessary scaffolding to help them uncover the underlying mathematical concepts. Throughout the lesson, the instructor offers continuous guidance and poses probing questions designed to stimulate higher-order thinking and strengthen problem-solving skills. Ultimately, this approach cultivates active participation and a shared sense of contribution among the class. The anticipated student conjectures for these activities are detailed in Table 2.

Table 2. Student Thinking Predictions in Activity 2

No	Activity	Expected Student Thinking & Informal Strategies	Potential Errors & Misconceptions	Teacher's Scaffolding / Intervention
1	Engage with the context-based video about *Lawang Sewu*	Boundary Tracing: Students visually identify and trace the outer frames of doors, windows, or courtyard spaces. Additive Thinking: Students recognize that finding the total length involves tracking the perimeter outline step-by-step.	Incorrectly Counting Sides: Due to perspective angles or complex overlapping architectural designs in the video, students might miss a side or double-count a shared boundary edge.	Provide static, clear close-up images of the *Lawang Sewu* structures.
2	Perform Activity 2 (Perimeter Calculation and Problem Solving)	Sequential Addition (Informal): Students calculate the perimeter by adding all four side lengths one by one (s + s + s + s). Proportional Reasoning: Students notice that since a square has equal sides, doubling two sides (2s + 2s) yields the total boundary. Generalization: Students transition to the multiplicative formula (4s).	Perimeter-Area Confusion: Students fail to differentiate the boundary from the interior space, inadvertently multiplying the side lengths (s × s) instead of adding them. Failure to Generalize Repeated Lengths: Students remain stuck in tedious addition, failing to recognize that the repetition of four identical lengths can be generalized into a more efficient multiplication formula (4s).	For Perimeter-Area Confusion: Use a "walking the fence" analogy: *"If an ant walks completely around this square window, does it walk across the glass (area) or just along the wooden frame (perimeter)?"* For Failure to Generalize: Prompt students to find a shortcut: *"You just added the number 5 four times. Is there a faster mathematical operation we can use when adding the exact same number repeatedly?"*

As outlined in Table 2, the HLT for learning perimeter in Activity 2 specifically anticipates three prominent student difficulties: tactical errors, such as counting sides incorrectly because of visual orientation; conceptual errors, such as confusing perimeter with area; and limited algebraic abstraction, manifested as a failure to generalize repeated side lengths into the formal 4s formula. By embedding structured, question-based scaffolding to address these specific obstacles, the framework ensures a robust transition from informal boundary tracing to formal geometric reasoning.

Reflecting on Activity 2, students are expected to meet the learning objectives related to the perimeter of a square. For students who encounter difficulties, the teacher provides targeted scaffolding. The instructor plays a vital role in stimulating learners by guiding them to synthesize the discoveries and information they gather as they construct the conceptual framework of squares.

The primary instructional goal of Activity 3 is to guide students in understanding and determining the area of a square. This session begins with a video presentation featuring the Lawang Sewu context, which serves as a visual aid to help students accurately perceive and describe square shapes through its architectural elements. Following this contextual introduction, students engage in a hands-on exploration using origami paper to conceptualize area as covering space.

Rather than treating the origami paper merely as a calculated object, students use smaller unit squares to physically cover the entire surface of the larger origami paper. This activity explicitly supports students' spatial reasoning by demonstrating that area represents the measure of a fully enclosed, continuous region. Progressively, this hands-on manipulation moves students away from counting individual unit squares one by one. By organizing the unit squares into a structured grid pattern on the origami paper, students are guided to recognize the row-and-column array. This spatial structure allows them to transition from primitive additive strategies, such as counting units or skip-counting rows, toward multiplicative reasoning, in which they discover that the total covered space can be calculated efficiently by multiplying the number of units along the length and width (side × side). Examples of the square motifs in Lawang Sewu's architecture are shown in Figure 3.

Figure 3. The repetitive square floor patterns and structural column symmetries in the *Lawang Sewu* corridor as a contextual foundation for horizontal mathematization

As illustrated in Figure 3, the visual material serves as more than a cultural artifact; it provides a structured mathematical context through the repetitive grid of its floor tiles and the symmetrical design of the corridor pillars. Mathematically, these floor patterns embed key geometric features of a square, such as four equal sides, orthogonal intersections (right angles), and a regular tessellation property.

In the learning trajectory, students use this visual prompt to bridge informal observation with formal geometric understanding through two progressive stages. First, the repeating nature of the floor tiles provides a spatial visualization tool for area exploration through informal unit-square tiling (row-and-column structure). Students can count or skip-count the tiles within a designated corridor section, which naturally transitions their thinking from additive strategies to multiplicative concepts (side × side). Second, by observing the square structures from a perspective angle along the hallway, students are guided to confront orientation and perspective biases. This helps them abstract the critical attributes of a square, namely equal side lengths and right angles, which remain constant regardless of the viewing angle, thereby elevating their geometric reasoning.

In this practical exercise, students are tasked with determining area using a combination of sticks and origami paper. Specifically, they are asked to calculate area using a given number of sticks as spatial measuring units. Following this tactile activity, learners explore the concept of area dynamically using GeoGebra. Drawing the square and determining its area at this stage are generally straightforward, enabling students to derive the area formula independently.

Nevertheless, some students may still make conceptual or calculation errors; in such instances, the teacher provides appropriate scaffolding. Once students successfully determine the area, the activity transitions toward applying these newly developed concepts to solve related contextual problems.

Throughout the lesson, the teacher continually supports students by posing thought-provoking questions designed to develop their higher-order thinking skills, particularly in solving geometric problems. This pedagogical approach not only fosters a highly interactive classroom but also encourages active student contributions to the learning environment. Furthermore, the students' anticipated cognitive processes and conjectures during these tasks are comprehensively outlined in Table 3.

Table 3. Student Thinking Predictions in Activity 3

No	Activity	Expected Student Thinking & Informal Strategies	Potential Errors & Misconceptions	Teacher's Scaffolding / Intervention
1	Engage with the context-based video about *Lawang Sewu*	Visual Recognition Students identify square patterns on floor tiles, windows, or door structures shown in the video. Informal Description: Students describe squares using non-formal attributes (for example, "all sides look equally long" or "it stands upright").	Orientation Bias: Students fail to recognize a square if it is captured from a skewed angle in the video, mistaking it for a rhombus or general quadrilateral. Over-generalization: Students might label non-square rectangular elements as squares due to low visual precision.	Provide freeze-frame images from the video and guide students to analyze the structural properties using guiding questions (for example, *"If we measure the adjacent sides of this window, what do you notice?"*).
2	Perform Activity 3	Tiling Strategy (Informal): Students	Perimeter-Area Confusion:	For Perimeter-Area Confusion: Ask students

No	Activity	Expected Student Thinking & Informal Strategies	Potential Errors & Misconceptions	Teacher's Scaffolding / Intervention
		manually count individual grid squares or floor tiles inside the designated area one by one. Additive/Skip Counting: Students count grid squares row-by-row or column-by-column (for example, 5, 10, 15...) Multiplicative Generalization: Students transition to multiplying the number of rows by the number of columns (side × side).	Students confuse the boundary with the surface space, calculating the area by adding the four outer sides (s+s+s+s or 4s) instead of multiplying them. Formula Blindness: Students plug numbers into the area formula without understanding spatial structures, leading to errors when encountering contextual word problems.	to shade the area they are calculating or ask: *"If you want to pave this floor with ceramic tiles, are you covering the borders or the whole surface?"* For Tiling Strategy: Encourage shift toward efficiency: *"Is there a faster way to find the total number of tiles without counting them one by one?"* to prompt the row-by-column multiplication concept.

As shown in Table 3, the learning trajectory in Activity 3 anticipates that students will not immediately employ formal geometric formulas. Instead, they are expected to move from informal strategies, such as visual noticing and unit-square tiling, toward formal multiplicative reasoning. By predicting potential misconceptions, such as confusing perimeter with area, the framework embeds proactive pedagogical scaffolding to ensure that the teacher can guide students' cognitive shifts dynamically during classroom experimentation.

Reflecting on Activity 3, students identify square formations and conduct physical experiments using sticks and origami paper, which are then complemented by dynamic digital exploration through GeoGebra to calculate area. After mastering these spatial concepts, learners apply their newly acquired knowledge to solve contextual problems. Students are expected to meet the instructional objectives of Activity 3. For those who encounter difficulties, the teacher provides targeted scaffolding. Throughout this process, the teacher acts as both a facilitator and a mediator, stimulating students' cognitive construction of square concepts.

Based on the presented findings, the preliminary phase of the design research involved preparing for the teaching experiment through several core activities. These included conducting a comprehensive literature review, analyzing the students' current and target competencies, selecting a relevant ethnomathematical context, formulating the Hypothetical Learning Trajectory, and creating innovative instructional media utilizing sticks, origami paper, and GeoGebra. These preparatory steps served as the essential foundation for developing the initial instructional strategy. As noted by Nursyahidah &

Albab (2021), a literature review plays a crucial role in identifying effective baseline strategies for formulating an HLT. This preliminary design stage is highly critical for transitioning students' understanding of quadrilaterals from informal, intuitive ideas to formal mathematical definitions. Furthermore, integrating cultural contexts within the TE-RME framework provides a meaningful starting point and serves as a valuable learning resource for mastering these concepts (Nursyahidah et al., 2025). Previous studies corroborate that instruction incorporating local cultural elements or indigenous knowledge familiar to the students significantly enhances their active engagement (Nursyahidah et al., 2025), conceptual understanding (Hardiyanto et al., 2024), and versatile problem-solving skills (Nursyahidah et al., 2025c).

The proposed instructional design is intentionally structured around the five core characteristics of Realistic Mathematics Education outlined by Putri & Zulkardi (2018), ensuring each principle is directly operationalized within the HLT activities: (1) The Use of Contexts: Rather than serving as a mere introductory backdrop, the ethnomathematical context of the Lawang Sewu heritage site in Semarang is deeply embedded in Activity 1 and Activity 2. Students confront real-world spatial scenarios through video and visual observations of the site's historical architecture, using these familiar, concrete structures to initiate their geometric inquiry into identifying squares and exploring boundaries. (2) The Use of Models: To bridge the gap between students' informal spatial reasoning and formal mathematical concepts, the design utilizes progressive modeling across the activities. In Activity 2, the physical arrangement of identical sticks serves as a concrete model-of a square's perimeter, which is later abstracted into a model-for dynamic covariation using GeoGebra sliders. Similarly, in Activity 3, the grid pattern of Lawang Sewu's floor tiles and the hands-on covering of origami paper serve as a model-of a continuous surface, progressing into a model-for abstract multiplicative area reasoning (side × side). (3) Student Contributions: The HLT shifts production entirely to the learners, making student-generated strategies the primary pathway to achieving the objectives. In the area and perimeter exploration phases (Activity 2 and Activity 3), the trajectory explicitly anticipates and utilizes diverse student productions, ranging from informal unit-counting and additive skip-counting to non-formal stick arrangements, as the core material for classroom mathematical development and formula construction. (4) Interactivity: Social interaction is deliberately engineered into the design of each activity. During the transition from visual notice (Activity 1) to physical perimeter modeling (Activity 2) and origami covering (Activity 3), the learning process mandates peer discussions and reciprocal explanations. Students must collectively negotiate feedback and collaborate to resolve predictable cognitive conflicts, such as why boundary-counting (perimeter) differs fundamentally from surface-covering (area). (5) Intertwining (Interconnectedness): The instructional activities avoid isolated skill drills by seamlessly intertwining interrelated mathematical domains and cultural heritage throughout the trajectory. In Activity 2 and Activity 3, core arithmetic operations (addition and multiplication) are directly integrated with geometric concepts (perimeter, area, and spatial tessellation) while analyzing the Lawang Sewu structures, thereby promoting a holistic and connected educational experience

Employing ethnomathematics as an educational context effectively stimulates both conceptual comprehension and problem-solving capabilities (Nursyahidah, 2021; Nursyahidah et al., 2025b). To reinforce the relevance and meaningfulness inherent in RME, every learning activity is intentionally designed to address real-life problems involving square shapes. Consequently, students are encouraged to participate actively and apply their mathematical discoveries to practical situations.

The integration of technology, specifically contextual videos and GeoGebra, further enhances students' motivation and active classroom involvement. Using contextual videos helps learners visually translate cultural contexts into abstract mathematical representations (Hidayat & Firmanti, 2024). Moreover, video-based learning simplifies complex concepts by engaging multiple senses simultaneously and creating a richer cognitive experience (Noetel et al., 2021). In parallel, GeoGebra empowers students to explore the geometric principles under investigation consciously and actively (Suparman et al., 2024). The application of GeoGebra has also been shown to increase learning motivation, deepen conceptual mastery (Zakaria et al., 2024), and foster robust problem-solving skills (Larico et al., 2026).

Ultimately, the TE-RME framework offers a comprehensive conceptual methodology for teaching quadrilaterals, particularly squares. By integrating cultural backgrounds into realistic scenarios, this approach holds the potential to enhance students' conceptual knowledge, engagement, and problem-solving proficiency. These anticipated outcomes align closely with prior studies, underscoring TE-RME's promise as a forward-thinking, impactful pedagogical strategy.

CONCLUSION

This study developed a Hypothetical Learning Trajectory (HLT) to facilitate the learning of quadrilaterals, with a specific focus on squares. This instructional design employed the Techno-Ethno-Realistic Mathematics Education (TE-RME) paradigm by integrating Lawang Sewu's architectural heritage in Semarang as a cultural anchor to support students' conceptual understanding. Formulated during the preliminary design stage, the proposed HLT is ready for subsequent empirical testing in the design experiment phase. Three primary tasks drive the instructional sequence: analyzing a Lawang Sewu video to deduce the geometric properties of a square, calculating its perimeter, and determining its area. This progressive arrangement was deliberately designed to foster deep conceptual mastery. Overall, the findings highlight the pedagogical potential of synergizing digital tools with local cultural insights through the TE-RME framework to enhance geometry instruction.

Notwithstanding these positive indications, several limitations of the study must be acknowledged. Primarily, the reliance on Lawang Sewu as a highly localized context may restrict the immediate transferability of the instructional design to learners from different cultural or geographical backgrounds. Additionally, the scope of the investigation was confined to conceptual understanding and problem solving related to squares, leaving other cognitive or affective dimensions unexamined. Addressing these methodological boundaries in subsequent studies will be essential for more robust validation of the approach.

Consequently, future research should scale the implementation of this HLT across more diverse educational settings. To accommodate demographic heterogeneity, scholars are encouraged to explore alternative ethnomathematical contexts and emerging technologies, thereby diversifying the available learning trajectories. Expanding the application of the TE-RME method to other mathematical domains and competence levels would also be highly beneficial. Such progressive inquiries will empower researchers and practitioners to continuously refine mathematics instruction through strategies that are both technologically progressive and culturally resonant.

ACKNOWLEDGMENT

The authors would like to extend their sincere gratitude to the Lembaga Penelitian dan Pengabdian kepada Masyarakat (LPPM) of Universitas PGRI Semarang for their financial support through the Regular Research Grant (023/SKK/LPPM-UPGRIS/REG/II/2026). The authors also thank the reviewers for their valuable feedback and support for this research paper.

REFERENCES

Amaliyah, Y., & Marsigit, M. (2024). Ethnomathematical analysis of student activities in associating quadrilateral and triangle concepts. Journal of Honai Math, 7(1), 57–70. https://doi.org/10.30862/jhm.v7i1.444

Amelia, R., & Herman, T. (2025). The effectiveness of hypothetical learning trajectory on quadrilateral topics for junior high school students. Journal of Innovative Mathematics Learning, 8(2), 242–252. https://doi.org/10.22460/jiml.v8i2.27044

Angraini, L. M., Susilawati, A., Noto, M. S., Wahyuni, R., & Andrian, D. (2024). Augmented reality for cultivating computational thinking skills in mathematics completed with literature review, bibliometrics, and experiments for students. Indonesian Journal of Science and Technology, 9(1), 225–260. https://doi.org/10.17509/ijost.v9i1.67258

Bakker, A. (2018). Design research in education: A practical guide for early career researchers. Routledge. https://doi.org/10.4324/9780203701010

Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 420–464). Macmillan.

Disnawati, H., Indra Putri, R. I., & Hartono, Y. (2012). Eksplorasi pemahaman siswa dalam pembelajaran bangun datar segi empat di SD menggunakan konteks Cak Ingkling. PYTHAGORAS Jurnal Matematika dan Pendidikan Matematika, 7(2), 65–80. https://doi.org/10.21831/pg.v7i2.4781

Fujita, T., & Jones, K. (2007). Learners’ understanding of the definitions and hierarchical classification of quadrilaterals: Towards a theoretical framing. Research in Mathematics Education, 9(1), 3–20. https://doi.org/10.1080/14794800008520167

Gravemeijer, K., & Cobb, P. (2006). Design research from a learning design perspective. In J. van den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), Educational design research (pp. 45–85). Routledge. https://doi.org/10.4324/9780203088364-12

Hardiyanto, D., Asokawati, I., Majid, P. M., Maesaroh, A. T., & Nursyahidah, F. (2024). Learning reflection using realistic mathematics education assisted by GeoGebra software. Jurnal Pendidikan Matematika, 18(1), 15–26. https://doi.org/10.22342/jpm.v18i1.pp15-26

Hidayat, A., & Firmanti, P. (2024). Navigating the tech frontier: A systematic review of technology integration in mathematics education. Cogent Education, 11(1), Article 2373559. https://doi.org/10.1080/2331186X.2024.2373559

Larico, D. V., Benavente Gutierrez, M. A., Talavera-Mendoza, F., & Rucano Paucar, F. H. (2026). Digital applications in mathematics learning for secondary school students: A systematic literature review. Eurasia Journal of Mathematics, Science and Technology Education, 22(1), Article em2768. https://doi.org/10.29333/ejmste/17760

Lipták, J. (2025). Integrating realism in mathematical problem solving: Insights from Stand and Deliver. Mathematics Teaching Research Journal, 17(5), 262–281.

National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Author.

Noetel, M., Griffith, S., Delaney, O., Sanders, T., Parker, P., del Pozo Cruz, B., & Lonsdale, C. (2021). Video improves learning in higher education: A systematic review. Review of Educational Research, 91(2), 204–236. https://doi.org/10.3102/0034654321990713

Nursyahidah, F., & Albab, I. U. (2021). Learning design on surface area and volume of cylinder using Indonesian ethno-mathematics of traditional cookie maker assisted by GeoGebra. Mathematics Teaching Research Journal, 13(4), 79–98.

Nursyahidah, F., Albab, I. U., & Rubowo, M. R. (2023). Learning design of sphere using realistic mathematics education assisted by interactive video. Jurnal Pendidikan Matematika, 17(3), 297–312. https://doi.org/10.22342/jpm.17.3.20040.297-312

Nursyahidah, F., Albab, I. U., & Rubowo, M. R. (2025). Designing learning trajectory on data distribution measurement through PMRI. Mathematics Education Journal, 19(3), 527–546. https://doi.org/10.22342/mej.v19i3.pp527-546

Nursyahidah, F., Anindya, F. M., Yulianti, M. A., Prisanto, Z. I., & Rosario, M. A. R. (2025). Integrating local wisdom with technology: Designing learning trajectory of cylinder through realistic mathematics education approach. Jurnal Pendidikan Matematika, 19(1), 81–98. https://doi.org/10.22342/jpm.v19i1.pp81-98

Nursyahidah, F., Wardono, Mariani, S., & Wijayanti, K. (2025a). Innovative learning techno-ethno-realistic mathematics education. Ganesha Kreasi Semesta.

Nursyahidah, F., Wardono, Mariani, S., & Wijayanti, K. (2025b). Integrating technology, Javanese ethnomathematics, and realistic mathematics education in supporting prospective mathematics teachers' numeracy skills: A learning trajectory. Journal on Mathematics Education, 16(2), 671–688. https://doi.org/10.22342/jme.v16i2.pp671-688

Nursyahidah, F., Wardono, Mariani, S., & Wijayanti, K. (2025c). Integrating technology, ethnomathematics, and realistic mathematics education in learning statistics: A learning trajectory. Infinity Journal, 14(3), 633–654. https://doi.org/10.22460/infinity.v14i3.p633-654

Nurtriana, N. A. I., & Nuryadi. (2025). The effectiveness of mathematics learning with the ethnomatematics approach to understanding junior high school students' concepts. Jurnal Mercumatika: Jurnal Penelitian Matematika dan Pendidikan Matematika, 8(2). https://doi.org/10.26486/jm.v8i2.4526

Prahmana, R. C. I., & D'Ambrosio, U. (2020). Learning geometry and values from patterns: Ethnomathematics on the batik patterns of Yogyakarta, Indonesia. Journal on Mathematics Education, 11(3), 439–456. https://doi.org/10.22342/jme.11.3.12949.439-456

Putri, R. I. I., & Zulkardi. (2018). Learning fraction through the context of Asian Games 2018. Journal of Physics: Conference Series, 1088, Article 012023. https://doi.org/10.1088/1742-6596/1088/1/012023

Ramadhan, F., Mahmudi, A., & Nabilla, H. A. (2025). The effectiveness of GeoGebra-assisted realistic mathematics education in enhancing students' conceptual understanding. Edumatica: Jurnal Pendidikan Matematika, 15(3), 402–416. https://doi.org/10.22437/edumatica.v15i3.45514

Salsabila, E., & Hajizah, M. N. (2023). Hypothetical learning trajectory of sector area and arc length using the clockwork context. PYTHAGORAS Jurnal Pendidikan Matematika, 18(2), 176–186. https://doi.org/10.21831/pythagoras.v18i2.67027

Sulistyowati, D., & Khotimah, R. P. (2022). An exploration of ethnomathematics at Sewu Temple in Yogyakarta. Jurnal Riset Pendidikan Matematika, 9(2), 177–190. https://doi.org/10.21831/jrpm.v9i2.51756

Suparman, S., Marasabessy, R., & Helsa, Y. (2024). Fostering spatial visualization in GeoGebra-assisted geometry lesson: A systematic review and meta-analysis. Eurasia Journal of Mathematics, Science and Technology Education, 20(9), Article em2509. https://doi.org/10.29333/ejmste/15170

Yuliardi, R., & Rosjanuardi, R. (2021). Hypothetical learning trajectory in student's spatial abilities to learn geometric transformation. JRAMathEdu: Journal of Research and Advances in Mathematics Education, 6(3), 174–190. https://doi.org/10.23917/jramathedu.v6i3.13338

Zakaria, M. I., Wong, W. S. C., Hanid, M. F. A., Adnan, M. F., Raimi, N. F., & Syed Azman, S. M. (2024). Integrating geometrical design with GeoGebra: Effects on motivation and academic performance among secondary students. Mathematics Teaching Research Journal, 16(5), 186–217.