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Learning Obstacles in Senior High School Students' Conceptual Understanding of Translation

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ABSTRACT

Students' difficulties in understanding translation indicate that learning problems in geometric transformation are not limited to procedural errors but are also related to obstacles in constructing and applying conceptual knowledge. This study aimed to analyze learning obstacles associated with senior high school students' conceptual understanding of translation using ontogenic, epistemological, and didactical obstacles as the analytical framework. The study employed a qualitative phenomenological approach as the initial phase of Didactical Design Research (DDR), focusing on the identification of learning obstacles before the development of a didactical design. The participants were 29 eleventh-grade students from a senior high school in Bandung, Indonesia, selected purposively based on variations in their solution and error patterns. Data were collected through four open-ended conceptual understanding test items, semi-structured interviews with selected students and a mathematics teacher, and documentation. Data analysis followed the stages of data reduction, data display, and conclusion drawing, while credibility was strengthened through triangulation of written responses, student interviews, and teacher interviews. The findings showed that students' conceptual understanding of translation remained limited across all indicators. The highest achievement was found in selecting and applying appropriate procedures or operations (41%), followed by representing concepts in various mathematical forms (34%), restating concepts (28%), and applying concepts or algorithms in problem-solving (14%). The analysis identified ontogenic instrumental obstacles related to insufficient mastery of Cartesian coordinates as prerequisite knowledge and epistemological obstacles related to students' limited ability to use translation concepts flexibly across visual, symbolic, procedural, and contextual representations. Epistemological obstacles appeared more dominant, whereas didactical obstacles could not be adequately identified from the available data. These findings provide an empirical basis for developing didactical designs that connect prerequisite knowledge, multiple representations, conceptual meaning, and contextual problem-solving.

ABSTRAK

Kesulitan siswa dalam memahami translasi menunjukkan bahwa permasalahan pembelajaran transformasi geometri tidak hanya berkaitan dengan kesalahan prosedural, tetapi juga dengan hambatan dalam membangun dan menerapkan pengetahuan konseptual. Penelitian ini bertujuan menganalisis hambatan belajar yang berkaitan dengan pemahaman konsep siswa SMA pada materi translasi dengan menggunakan hambatan ontogenik, epistemologis, dan didaktis sebagai kerangka analisis. Penelitian menggunakan pendekatan fenomenologi kualitatif sebagai tahap awal Didactical Design Research (DDR) yang berfokus pada identifikasi hambatan belajar sebelum pengembangan desain didaktis. Subjek penelitian terdiri atas 29 siswa kelas XI dari salah satu SMA di Kota Bandung, Indonesia, yang dipilih secara purposif berdasarkan variasi pola jawaban dan kesalahan siswa. Data dikumpulkan melalui empat soal uraian pemahaman konsep, wawancara semi-terstruktur dengan siswa terpilih dan seorang guru matematika, serta dokumentasi. Analisis data dilakukan melalui reduksi data, penyajian data, dan penarikan kesimpulan, sedangkan kredibilitas temuan diperkuat melalui triangulasi jawaban tertulis, hasil wawancara siswa, dan wawancara guru. Hasil penelitian menunjukkan bahwa pemahaman konsep siswa pada materi translasi masih terbatas pada seluruh indikator. Capaian tertinggi terdapat pada indikator memilih dan menerapkan prosedur atau operasi yang tepat sebesar 41%, diikuti kemampuan merepresentasikan konsep dalam berbagai bentuk matematis sebesar 34%, menyatakan kembali konsep sebesar 28%, serta menerapkan konsep atau algoritma dalam pemecahan masalah sebesar 14%. Analisis mengidentifikasi hambatan ontogenik instrumental berupa keterbatasan penguasaan koordinat Kartesius sebagai pengetahuan prasyarat dan hambatan epistemologis berupa ketidakmampuan siswa menggunakan konsep translasi secara fleksibel pada representasi visual, simbolik, prosedural, dan kontekstual. Hambatan epistemologis tampak lebih dominan, sedangkan hambatan didaktis belum dapat diidentifikasi secara memadai berdasarkan data yang tersedia. Temuan ini memberikan landasan empiris bagi pengembangan desain didaktis yang menghubungkan pengetahuan prasyarat, beragam representasi, makna konseptual, dan pemecahan masalah kontekstual.

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INTRODUCTION

Conceptual understanding is a fundamental aspect of mathematics learning because it provides a foundation for students to understand relationships among mathematical concepts, develop mathematical thinking skills, and apply their knowledge in various problem-solving situations (Diana et al., 2020; Khairani et al., 2024). Conceptual understanding enables students not only to memorize formulas or procedures but also to understand the meaning underlying mathematical concepts and to represent mathematical ideas in multiple forms (Cahyaningrum et al., 2023; Mutawah et al., 2019). Furthermore, the National Council of Teachers of Mathematics (NCTM, 2000) identifies conceptual understanding as a key component of mathematical proficiency that should be developed alongside procedural fluency, strategic competence, adaptive reasoning, and productive disposition.

One mathematical topic that requires strong conceptual understanding is geometric transformation (Ahmadi et al., 2024). Within this topic, translation involves moving a geometric object according to a specified direction and magnitude. Therefore, students are expected to understand the relationships among visual representations on the Cartesian plane, translation vectors, and the mathematical procedures used to determine the image of a translated object (Nurmitasari et al., 2023). These characteristics make translation a concept that requires not only computational skills but also conceptual understanding of the relationships among the pre-image, the translation vector, and the resulting image. Mastering the concept of translation is essential because it serves as a foundation for understanding other geometric transformations, such as reflection, rotation, and dilation, while also fostering students' spatial and mathematical reasoning skills.

Previous studies have consistently reported that students experience difficulties in understanding the concept of translation. These difficulties are reflected in students' inability to interpret a translation vector as a quantity with both direction and magnitude, errors in determining the coordinates of the translated image, and misconceptions regarding the roles of the x- and y-axes on the Cartesian plane (Nurrahmawati et al., 2021). Furthermore, students encounter difficulties in identifying and describing translations, applying translation procedures accurately, and using appropriate mathematical terminology (Azka et al., 2024; Ziliwu et al., 2022). These findings suggest that students' difficulties with translation involve several aspects of conceptual understanding, including restating mathematical concepts, representing concepts in multiple mathematical forms, selecting and applying appropriate procedures or operations, and applying concepts or algorithms in problem-solving. Therefore, students' difficulties in learning translation are not limited to procedural accuracy but also indicate limitations in constructing and applying coherent conceptual understanding.

Students' difficulties in understanding the concept of translation indicate that limitations in conceptual understanding are not merely associated with obtaining incorrect answers but also with obstacles encountered during the construction of mathematical knowledge. Conceptual understanding develops through the process of connecting mathematical ideas, multiple representations, and students' prior learning experiences. When this process is disrupted, students may experience difficulties interpreting concepts, relating different representations, and applying their knowledge across various contexts. From the perspective of Didactical Design Research (DDR), these difficulties can be analyzed through the concept of learning obstacles, which refers to factors that hinder students' learning processes and the construction of mathematical knowledge. According to Brousseau (2002), learning obstacles are classified into three categories: ontogenic obstacles, which are associated with students' readiness and prior capabilities; epistemological obstacles, which arise from limitations in applying knowledge across different contexts; and didactical obstacles, which are related to the ways in which instructional content is presented and learning activities are designed. Therefore, identifying learning obstacles in translation is essential for revealing the underlying factors that constrain students' conceptual understanding and for providing an empirical basis for designing instruction that is responsive to students' learning needs.

Research on geometric transformations has been extensively conducted, including studies on the development of technology-based instructional designs and learning media (Jelatu et al., 2018; Kholifah et al., 2023; Listiawan & Antoni, 2021; Rosyada et al., 2025), the improvement of students' spatial abilities (Febrianti et al., 2024), and the implementation of various learning approaches (Bustan et al., 2022). In addition, several studies have identified various forms of students' difficulties and misconceptions in learning translation (Mbusi & Luneta, 2021, 2023). However, these studies have generally focused on describing students' errors or improving learning outcomes, whereas investigations that analyze these difficulties as learning obstacles related to students' conceptual understanding of translation remain limited. In fact, analyzing learning obstacles is essential for uncovering the underlying causes of students' learning difficulties and providing a foundation for designing more effective learning experiences.

Based on the aforementioned discussion, this study aimed to analyze learning obstacles related to senior high school students' conceptual understanding of translation based on the classification of ontogenic, epistemological, and didactical obstacles as the analytical framework. The findings are expected to provide theoretical contributions by enriching the body of knowledge regarding learning obstacles in geometric transformation learning and to serve as a foundation for teachers in designing didactical designs that anticipate students' learning obstacles and facilitate more meaningful learning of translation.

METHOD

This study employed a qualitative phenomenological approach as the initial phase of the Didactical Design Research (DDR) framework developed by Suryadi (2025) to identify learning obstacles related to students' conceptual understanding of translation. The phenomenological approach was used to explore students' learning experiences in constructing the concept of translation, particularly how they interpreted representations, procedures, and the application of concepts when solving translation problems. This study did not include the development or implementation phases of a didactical design; instead, it focused on revealing the characteristics of learning obstacles that serve as the foundation for formulating subsequent didactical designs.

The participants consisted of 29 eleventh-grade students from a senior high school in Bandung, Indonesia, who were selected through purposive sampling. The selection of participants was based on several criteria: the students had learned translation according to the applicable curriculum, demonstrated variations in solution patterns based on the results of the conceptual understanding test, and were willing to participate in interviews to reveal their thinking processes. Data were collected through a conceptual understanding test consisting of four open-ended questions, semi-structured interviews with students and a mathematics teacher, and documentation. The test instrument was developed based on indicators of conceptual understanding adapted from Kilpatrick et al. (2001), namely restating concepts, selecting and applying appropriate procedures or operations, representing concepts in various forms of mathematical representation, and applying concepts or algorithms in problem-solving. The four questions were considered adequate because each question was designed to represent one indicator of conceptual understanding and enabled the researchers to analyze not only students' final answers but also the strategies and error patterns that emerged during the problem-solving process.

Data analysis was conducted using the Miles and Huberman (1984) model, which consists of data reduction, data display, and conclusion drawing. The analysis began by identifying students' error patterns based on each indicator of conceptual understanding. Subsequently, several students representing specific error patterns were selected for interviews to obtain information about their thinking processes, reasons for selecting particular strategies, and interpretations of the concept of translation. Interviews with the mathematics teacher were conducted as supporting data to obtain information about the translation learning process, the instructional strategies used in presenting the material, and instructional factors that may have contributed to students' difficulties.

The identification of learning obstacles was conducted through data triangulation by comparing students' written responses, student interview results, and information obtained from mathematics teacher interviews. The test data were used to identify students' error patterns, student interviews were used to clarify the thinking processes underlying those errors, and teacher interviews were used to provide contextual information about students' learning experiences. The comparison of these three data sources was subsequently analyzed based on Brousseau's (2002) learning obstacle framework and Suryadi's (2025) development of the framework. To maintain consistency in data interpretation, the classification of learning obstacles in this study employed the operational criteria presented in Table 1.

Table 1. Criteria for Identifying Learning Obstacles

Type of Learning Obstacle	Identification Criteria
Ontogenic Obstacle	Obstacles related to limitations in students' learning readiness or mastery of prerequisite knowledge required to understand the concept of translation.
Epistemological Obstacle	Obstacles that emerge when students' existing knowledge cannot yet be applied flexibly across different representations or contexts.
Didactical Obstacle	Obstacles related to the presentation of learning materials or instructional processes that may influence the construction of students' understanding.

Based on the criteria presented in Table 1, each pattern of students' difficulties was categorized according to the characteristics of the obstacles that most appropriately corresponded to the obtained data.

RESULTS AND DISCUSSION

Based on students' responses to the conceptual understanding test on translation, the percentage of achievement for each conceptual understanding indicator is presented in Table 2.

Table 2. Percentage of Achievement for Conceptual Understanding Indicators

Question Number	Conceptual Understanding Indicator	Number of Students	Percentage (%)
1	Restating a concept	8	28
2	Representing concepts in various forms of mathematical representation	10	34
3	Selecting, applying, and utilizing appropriate procedures or operations	12	41
4	Applying concepts or algorithms in problem-solving	4	14

Based on Table 2, the highest proportion of student achievement was observed in the indicator of selecting, applying, and utilizing appropriate procedures or operations, with a percentage of 41%. Although this indicator showed the highest percentage, the finding indicates that students' ability to use appropriate procedures or operations remained limited, as it was achieved by less than half of the students. Meanwhile, the lowest proportion was found in the indicator of applying concepts or algorithms in problem-solving, with a percentage of 14%. The low percentage for this indicator suggests that applying concepts to solve problems remained the most challenging aspect for students compared with other indicators of conceptual understanding. The notable difference between the indicator of selecting, applying, and utilizing appropriate procedures or operations and the indicator of applying concepts or algorithms in problem-solving suggests that students' procedural abilities had not yet been accompanied by the ability to apply conceptual knowledge in problem-solving contexts. To obtain a deeper understanding of the characteristics of students' conceptual understanding across each indicator, an analysis of students' responses to each test item is presented below.

Question 1, which was designed based on the conceptual understanding indicator of restating a concept, aimed to identify students' ability to restate the concept of translation. The question is presented in Figure 1.

Observe Figure 1 bellow:

(Figure 1)

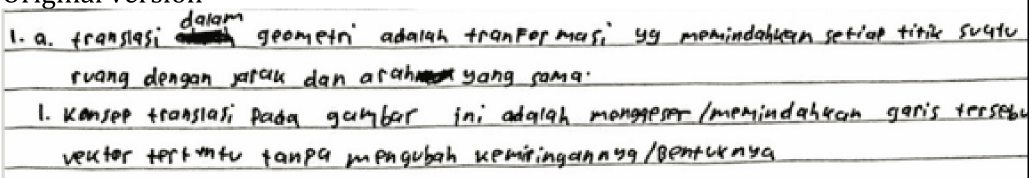
Based on Figure 1, answer the following questions:
a. What is meant by translation in geometry?
b. Explain the concept of translation based on Figure 1.

Figure 1. Question 1

Based on Figure 1, Question 1 was designed to assess students' ability to restate the concept of translation through a Cartesian coordinate representation. Through this question, students were expected to explain the meaning of translation by relating the original point, the image point, and the translation vector presented in the figure. Students' responses to Question 1 were analyzed to identify the extent to which they were able to understand and explain the concept of translation based on the given representation.

Based on the test results for Question 1, 28% of the students were able to restate the concept of translation as a transformation that moves the position of an object (point, line, or figure) by a certain distance and direction. In contrast, the majority of students were unable to accurately explain the concept of translation based on the visual representation provided on the Cartesian coordinate plane. This was indicated by students' responses that did not demonstrate the ability to relate their explanations of the translation concept to the information presented in the figure, such as the pre-image, image, and translation vector. One student's incorrect response to Question 1 is presented in Figure 2.

Original version


1. a. translasi dalam geometri adalah tranformasi yg memindahkan setiap titik suatu ruang dengan jarak dan arah yang sama.
1. konsep translasi pada gambar ini adalah menggeser/memindahkan garis tersebu vektor tertentu tanpa mengubah kemiringannya/bentuknya

English version:
1. a. Translation in geometry is a transformation that moves each point of an object or space by the same distance and in the same direction.
b. The concept of translation in this figure is shifting or moving an object by a certain vector without changing its slope or shape.

Figure 2. Example of S28's Incorrect Response to Question 1

Based on S28's response in Figure 2, S28 was able to restate the definition of translation as a transformation that moves every point of an object by the same distance and direction. However, S28 was not yet able to explain the concept of translation based on the visual representation provided in the question. This was indicated by S28's explanation, which remained general and did not relate the concept of translation to the elements presented in the figure, such as the original point (pre-image), the translated point (image), and the translation vector. Furthermore, S28 stated that the translated object was a "line," whereas the object shown in the figure was actually a point. To obtain deeper insights into S28's understanding, an interview was conducted with S28. The interview findings are presented in Table 3 to further support the identification of students' difficulties in understanding the concept of translation.

Table 3. Interview Results with S28 for Question 1

Question	S28’s Response
Why did you write in your answer sheet that the object being moved or translated was a line?	Because there is a line in the figure, Ma'am.
In the figure, there are points A, A', and vector T. In your opinion, what do they represent in the concept of translation?	A and A' are lines that are moved by vector T, Ma'am.
Do you know the original point or pre-image in the illustration in Figure 1?	I am not really sure, Ma'am.
How about the resulting point or image?	I am not really sure about that either, Ma'am.

Based on Table 3, the interview findings indicate that S28 considered the object undergoing translation to be a line because the student focused on the arrow line displayed in the figure. S28 was also unable to interpret the elements presented in the visual representation, as the points A and A' were understood as a line moved by vector T. Furthermore, S28 expressed a lack of understanding of the meaning of the original point (pre-image) and the translated point (image). These findings indicate that S28 was not yet able to relate the visual representation in the figure to the appropriate concept of translation.

The interview findings with S28 were subsequently triangulated through an interview with the mathematics teacher to obtain information about students' tendencies when solving translation problems during the learning process. The results of the teacher interview are presented in Table 4.

Table 4. Teacher Interview Results Regarding Students' Difficulties in Learning Translation

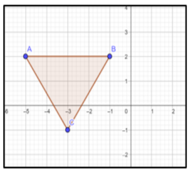
Question	Teacher's Response
What are students' tendencies when solving translation problems during the learning process?	When given practice problems, students tend to focus more on the answer than on understanding the process or identifying the problem first.
Are students able to understand the purpose of the given problem?	When asked what the problem was or what the purpose of the question was, they were also unable to answer.
What difficulties are most frequently experienced by students?	Students can add the vector components to the coordinates of a point, but when they are asked what point A, A', image, or pre-image means, many of them cannot explain the meaning.

Based on Table 4, the teacher interview findings indicate that some students tended to focus on obtaining answers rather than understanding the underlying concepts involved in solving problems. The teacher explained that students were generally able to apply translation procedures but still experienced difficulties explaining the meaning of the elements involved in the concept of translation, such as the original point (pre-image), the translated point (image), and the purpose of solving the problem. These findings are consistent with the interview results from S28, which showed that the student was unable to interpret the visual representation in the figure as part of the concept of translation.

Based on the triangulation of test results, student interviews, and teacher interviews, it can be concluded that students had not yet fulfilled the conceptual understanding indicator of restating a concept. This difficulty was demonstrated by students' inability to explain the concept of translation based on the visual representation presented on the Cartesian coordinate plane. These findings indicate the presence of an epistemological obstacle, namely an obstacle arising from students' limited understanding of the concept of translation, in which their knowledge remained restricted to a general definition and could not yet be applied to interpret visual representations in different contexts.

Question 2, which was designed based on the conceptual understanding indicator of representing concepts in various forms of mathematical representation, aimed to identify students' ability to represent translation from visual representations to symbolic forms. The question is presented in Figure 3.

2. Observe Figure 1 bellow:



(Figure 2)

Based on Figure 2, determine the image of triangle ABC translated by the $T = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$

Figure 3. Question 2

Based on Figure 3, Question 2 was designed to assess students' ability to determine the coordinates of the translated image of the vertices of triangle ABC based on the visual representation on the Cartesian coordinate plane and the given translation vector. Students' responses were analyzed to identify their ability to accurately transform visual representations into symbolic representations.

(x, y) Based on the test results for Question 2, 34% of the students were able to represent the concept of translation in a mathematical representation accurately. In contrast, the majority of students were unable to transform the visual representation on the Cartesian coordinate plane into a symbolic representation in the form of point coordinates and translated images. These difficulties were indicated by students' inability to accurately read point coordinates, including errors in determining the order of coordinate pairs and errors in identifying positive and negative signs in the coordinates of points A, B, and C. One student's incorrect response to Question 2 is presented in Figure 4.

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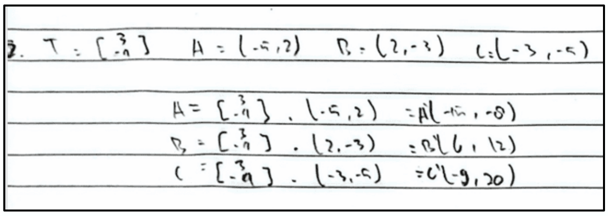


Figure 4. Example of S17's Incorrect Response to Question 2

$(2,-3)(-3,-5)(-1,2)(-3,-1)$Based on S17's response in Figure 4, S17 was unable to accurately read the coordinates of points on the Cartesian coordinate plane. This was indicated by errors in determining the coordinates of points B and C, where S17 identified the coordinates of point B and point C incorrectly. These errors indicate that the symbolic representation produced by S17 was not consistent with the visual representation presented on the Cartesian coordinate plane. To obtain deeper insights into these difficulties, an interview was conducted with S17. The interview results are presented in Table 5.

Table 5. Interview Results with S17 for Question 2

Question	S17's Response
Can you explain how you determined the coordinates of points A, B, and C?	Point A is here, Ma'am *(pointing to point A)*, so its coordinates are -5 *(moving the index finger from point A to point B)* and 2. Point B is this one *(pointing to point B)*, so its coordinates are 2 *(moving the index finger from point B to point C)* and -3. As for point C *(pointing to point C)*, its coordinates are -3 *(moving the index finger from point C to point A)* and -5.
Why did you determine the coordinates in that way?	I followed the sides of the triangle, Ma'am.
Why did you follow the sides of the triangle?	Because the points are connected by lines.

Based on Table 5, the interview findings indicate that S17 determined the coordinates of the points by following the line segments connecting the vertices of the triangle rather than by projecting each point onto the horizontal axis (x-axis) and the vertical axis (y-axis). As a result, the coordinates of a point were formed by combining the x- and y-coordinates of different points. For example, when determining the coordinates of point B, S17 used the y-coordinate of point B but paired it with the x-coordinate of point C because the two points were connected by one side of the triangle. These findings indicate that S17 had not yet understood how to read point coordinates on the Cartesian coordinate plane, which is a prerequisite for transforming visual representations into symbolic representations.

The interview findings with S17 were subsequently triangulated through an interview with the mathematics teacher to confirm students' difficulties related to prerequisite knowledge. The results of the teacher interview are presented in Table 6.

Table 6. Teacher Interview Results Regarding Students' Understanding of Prerequisite Knowledge

Question	Teacher's Response
Is students' understanding of the prerequisite knowledge sufficiently strong?	Some students still have a limited understanding of the prerequisite knowledge, particularly the concepts of matrix vectors and Cartesian coordinates.
What are these difficulties, and how do they affect students' understanding of the concept of translation?	Some students still have difficulty reading coordinates on the Cartesian coordinate plane. When the coordinates of the points are already provided in the question, they can solve the problem easily. However, when they have to determine the point coordinates first, they experience difficulties.

Based on Table 6, the teacher interview findings indicate that some students still experienced difficulties in reading coordinates on the Cartesian coordinate plane, which is a prerequisite for understanding translation. The teacher explained that students were generally able to solve translation problems when the point coordinates were provided directly. However, when students were required to determine the coordinates of points from a visual representation on the Cartesian coordinate plane, they experienced difficulties. These findings reinforce the results of the test and the interview with S17, suggesting that the difficulties encountered in Question 2 were related to students' insufficient mastery of reading Cartesian coordinates as a prerequisite for transforming visual representations into symbolic representations.

Based on the triangulation of the test results, student interviews, and teacher interviews, it can be concluded that students had not yet fulfilled the conceptual understanding indicator of representing concepts in various forms of mathematical representation. This difficulty was demonstrated by students' inability to transform the visual representation on the Cartesian coordinate plane into a symbolic representation in the form of point coordinates, which served as the basis for determining the translated image. These findings indicate the presence of an ontogenic instrumental obstacle associated with students' limited mastery of prerequisite knowledge required to read Cartesian coordinates for representing the concept of translation. Consequently, students' understanding of translation was not yet supported by the ability to accurately transform visual information into symbolic representations.

Question 3, which was designed based on the conceptual understanding indicator of selecting, applying, and utilizing appropriate procedures or operations, aimed to identify students' ability to select and apply appropriate procedures or operations in solving translation problems. The question is presented in Figure 5.

3. Point $C(-2,-1)$ is translated successively by the vectors $T = \begin{bmatrix} -3 \\ -3 \end{bmatrix}$, $U = \begin{bmatrix} 0 \\ 6 \end{bmatrix}, and\ V = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$. Determine the image of point C using matrix operations.

Figure 5. Question 3

Based on Figure 5, Question 3 was designed to assess students' ability to determine the image of point C after three successive translations by the vectors T, U, and V using matrix operations. Students' responses were analyzed to identify their ability to systematically apply the translation procedure, beginning with the first translation and continuing until the final image of the translated point was obtained.

Based on the test results for Question 3, 41% of the students were able to apply the translation procedure correctly. In contrast, the majority of students still experienced difficulties applying successive translation procedures in accordance with the underlying concept. One student's incorrect response to Question 3 is presented in Figure 6.

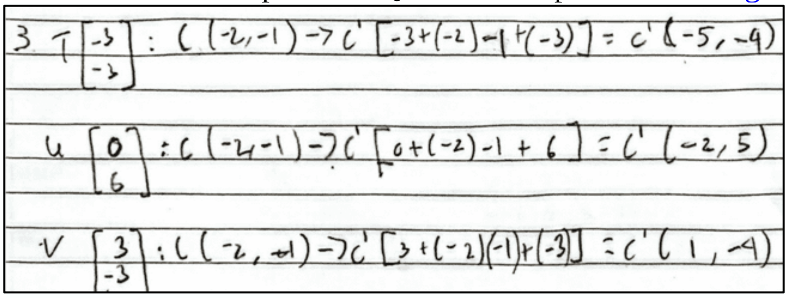


Figure 6. Example of S1's Incorrect Response to Question 3

Based on S1's response in Figure 6, S1 was unable to correctly apply the concept of successive translations. This was demonstrated by the way S1 treated the translations by vectors T, U, and V as three independent processes. Consequently, S1 translated point C separately using each vector, resulting in three different image points rather than using the result of the previous translation as the starting point for the next translation to obtain a single final image. To obtain deeper insights into this difficulty, an interview was conducted with S1. The interview results are presented in Table 7.

Table 7. Interview Results with S1 for Question 3

Question	S1's Response
What was being asked in Question 3?	The image of point C after being translated by T, U, and V three times successively.
How did you solve Question 3?	Since T, U, and V are already given, I just added C to T, U, and V.
What does "translated three times successively" mean?	Point C is first translated by T, then point C is translated by U, and finally by V, so there are three image points.

Based on Table 7, the interview findings indicate that S1 interpreted successive translations as separate translation processes applied to the initial point C. This understanding shows that S1 had not yet recognized that, in successive translations, the result of each translation becomes the starting point for the subsequent translation. Consequently, the procedure applied by S1 was inconsistent with the concept of successive translation, resulting in the inability to obtain the correct final image after the translation process.

The interview findings with S1 were subsequently triangulated through an interview with the mathematics teacher to obtain further information about students' understanding and difficulties. The results of the teacher interview are presented in Table 8.

Table 8. Teacher Interview Results Regarding Students' Understanding of the Concept of Translation Composition

Question	Teacher's Response
Regarding the concept of translation composition, do students also experience difficulties?	Many students still experience difficulties with the concept of translation composition. This may be because students are accustomed to solving single translations; therefore, when multiple translations are involved, they assume that all translations are performed from the initial point.

Based on Table 8, the teacher interview findings indicate that some students still experienced difficulties understanding the concept of successive translations. The teacher explained that students were accustomed to solving single-translation problems; therefore, when they encountered multiple translations performed sequentially, they tended to assume that each translation was directly applied to the initial point. As a result, students did not use the result of the previous translation as the starting point for the subsequent translation process. These findings are consistent with the interview results from S1, which showed that the student interpreted successive translations as several independent translation processes.

Based on the triangulation of test results, student interviews, and teacher interviews, it can be concluded that students had not yet fulfilled the conceptual understanding indicator of selecting, applying, and utilizing appropriate procedures or operations. This difficulty was demonstrated by the inappropriate use of successive translation procedures, in which each translation vector was directly applied to the initial point without using the result of the previous translation as the starting point for the next process. These findings indicate the presence of an epistemological obstacle arising from students' incomplete understanding of successive translations, which remained limited and resulted in the selection of procedures that were inconsistent with the underlying meaning of the concept.

Question 4, which was designed based on the conceptual understanding indicator of applying concepts or algorithms in problem-solving, aimed to identify students' ability to apply the concept of translation in a contextual problem. The question is presented in Figure 7.

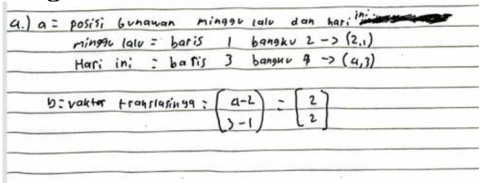
4. A Grade 11-I classroom has 4 rows of student desks, with 5 desks in each row. The rows are numbered starting from the front of the classroom, and the desks in each row are numbered from left to right. Last week, Gunawan sat at Desk 2 in Row 1. Today, he is sitting at Desk 4 in Row 3. Based on this information, answer the following questions:
   a. Illustrate Gunawan's seat positions last week and today on the Cartesian coordinate plane.
   b. Determine the translation vector that maps Gunawan's seat from its original position to its new position.

Figure 7. Question 4

Based on Figure 7, Question 4 was designed to assess students' ability to illustrate the positions of seats on the Cartesian coordinate plane based on the given information and subsequently determine the translation vector that moves the initial position to the final position. Students' responses were analyzed to identify their ability to represent contextual information in mathematical form and apply the concept of translation to solve the given problem.

Based on the test results for Question 4, 14% of the students were able to apply the concept of translation accurately in solving a contextual problem. In contrast, the majority of students still experienced difficulties representing contextual information according to the requirements of the problem. These difficulties were indicated by students' tendency to directly determine the coordinates of the initial and final positions, as well as the translation vector, without first illustrating the positions on the Cartesian coordinate plane. One student's incorrect response to Question 4 is presented in Figure 8.

Original version:



English version:
a. Gunawan's position last week and today
Last week = row 1, seat 2 (2, 1)
Today = row 3, seat 4 (4, 3)
b. The translation vector

Figure 8. Example of S18's Incorrect Response to Question 4

Based on S18's response in Figure 8, S18 did not illustrate the initial and final seat positions on the Cartesian coordinate plane as required by the problem. S18 directly represented the seat positions as coordinate pairs, namely (2, 1) for the initial position and (4, 3) for the final position, and then determined the translation vector by calculating the difference between the two coordinates. Although the obtained translation vector was correct, the solution did not fully meet the requirements of the problem because the student did not provide a visual representation on the Cartesian coordinate plane before determining the translation vector. To obtain deeper insights into this solution process, an interview was conducted with S18. The interview results are presented in Table 9.

Table 9. Interview Results with S18 for Question 4

Question	S18's Response
What was being asked in Question 4?	The illustration of the seat positions last week and today on the Cartesian coordinate plane, as well as the translation vector.
How did you determine the coordinate points for the positions last week and today?	The initial point is at seat number 2 in row 1, so it is (2, 1). The image is at seat number 4 in row 3, so it is (4, 3).
Why could the order be written that way?	Based on Question 4, because the seat number is mentioned before the row ("desk 2 in row 1" and "desk 4 in row 3" in the problem).
If the row were mentioned before the seat number, would the order of the coordinates change?	It would change according to the order of mention.

Based on Table 9, the interview findings indicate that S18 directly represented the seat positions in the form of coordinate pairs without first illustrating them on the Cartesian coordinate plane as required by the problem. S18 explained that the coordinate pairs were determined based on the order in which the information was mentioned in the problem, namely “seat” followed by “row.” This was further confirmed when S18 stated that the order of the coordinates would change if the order of the information in the problem also changed. These findings indicate that S18 constructed the coordinate representation based on the sequence of information provided in the problem rather than on the spatial relationships described in the problem context. In addition, S18 revealed that he/she had forgotten how to determine the position of a point on the Cartesian coordinate plane, which resulted in the student not illustrating the seat positions as required by the problem.

Based on the test results and student interviews, it can be concluded that students had not yet fulfilled the conceptual understanding indicator of applying concepts or algorithms in problem-solving. This difficulty was demonstrated by students' inability to construct a mathematical model by determining point coordinates based on contextual information, resulting in solutions that did not fully meet the requirements of the problem. The interview findings showed that students determined coordinate pairs based on the order in which information was mentioned in the problem rather than on the spatial relationships described in the problem context. These findings indicate the presence of an epistemological obstacle arising from students' inability to interpret contextual information as a basis for constructing mathematical models and applying the concept of translation in problem-solving.

The findings of this study indicate that students' conceptual understanding of translation had not yet developed optimally across all indicators. The pattern of achievement suggests that students' difficulties did not only emerge at the stage of applying concepts in problem-solving but also during the processes of understanding concepts, representing information, and selecting solution procedures. Therefore, obstacles in understanding the concept of translation are not singular in nature but occur across several interrelated stages that influence students' ability to solve increasingly complex tasks.

For the indicator of restating a concept, some students were able to state the definition of translation verbally but were unable to explain the concept based on the visual representation provided. This indicates that students' understanding of translation had not yet developed comprehensively because they were unable to connect the conceptual definition with the visual representation on the Cartesian coordinate plane. According to Kilpatrick et al. (2001), conceptual understanding is not only demonstrated through the ability to recall or restate a concept but also through the ability to understand the meaning of a concept and establish connections among various mathematical representations. Therefore, in this study, the ability to restate a concept is not merely interpreted as the ability to state the definition of translation, but also as the ability to explain the meaning of the concept based on the given representation. This finding is consistent with the study by Hafizh and Kusno (2025), which showed that students tend to understand geometric transformations procedurally and verbally but experience difficulties when concepts are presented in visual representations.

For the indicator of representing concepts in various forms of mathematical representation, some students still experienced difficulties converting visual representations on the Cartesian coordinate plane into symbolic representations in the form of point coordinate pairs. According to Duval (2006), mathematical understanding is not only determined by the ability to use a particular representation but also by the ability to convert between different forms of representation to represent the same concept. Therefore, students' difficulties in determining point coordinates indicate that the conversion process from visual to symbolic representations had not been carried out appropriately, resulting in the information presented on the Cartesian coordinate plane not being accurately represented in coordinate form. This finding is consistent with Cahyaningrum et al. (2023), who reported that students still experience difficulties connecting visual and symbolic representations in geometric transformation topics, leading to frequent errors in determining the coordinates of transformed objects.

For the indicator of selecting, applying, and utilizing appropriate procedures or operations, students still experienced difficulties selecting appropriate procedures to solve the composition of translations. The findings indicate that students applied each translation vector directly to the initial point, resulting in procedures that were inconsistent with the meaning of successive translations as a sequence of interconnected transformations. Procedural knowledge is related to the mastery of solution steps, whereas conceptual knowledge refers to understanding the relationships among mathematical ideas underlying those procedures. Based on this perspective, the errors identified in this study not only demonstrate inaccuracies in selecting procedures but also indicate that students had not yet developed an understanding of the concept of composition of translations underlying the use of such procedures. This finding is consistent with the studies by Ahmadi et al. (2024) and Sunariah and Mulyana (2020), which showed that procedural errors in geometric transformation topics often stem from a lack of conceptual understanding of transformation concepts.

For the indicator of applying concepts or algorithms in problem-solving, students still experienced difficulties applying the concept of translation to contextual problems. The findings indicate that students were unable to construct appropriate mathematical models based on the given information, resulting in the concept of translation not being applied through mathematical representations developed from the problem context. According to NCTM (2000), problem-solving in mathematics learning does not only require the ability to apply procedures but also the ability to interpret information, represent situations mathematically, and apply appropriate concepts to obtain solutions. Therefore, in this study, the ability to apply the concept of translation is not only demonstrated through determining the translation vector but also through the ability to construct appropriate mathematical representations based on the problem context before applying the concept. This finding is consistent with the study by Tsabita Primrose and Masamah (2025), which showed that students still experience difficulties translating contextual information into mathematical models in geometric transformation topics.

Based on the analysis of each conceptual understanding indicator, students' difficulties in understanding translation not only reflect limitations in mastering the concept but also reveal different characteristics of learning obstacles. Therefore, to obtain a more comprehensive understanding of the factors underlying these difficulties, further analysis was conducted to identify the types of learning obstacles based on the classification proposed by Brousseau (2002) and further developed by Suryadi (2025).

In this study, the analysis of learning obstacles focused on obstacles that could be identified based on data regarding students' understanding obtained from test results, student interviews, and teacher interviews. Based on these data, the findings revealed the presence of ontogenic and epistemological obstacles in students' understanding of the concept of translation. Meanwhile, didactical obstacles could not be adequately identified because the research data did not provide sufficient information about the influence of material presentation, learning sequences, or previous didactical strategies on the emergence of students' difficulties. Therefore, this study does not conclude that didactical obstacles do not exist; rather, it indicates that the obstacles identified within the research context were more closely related to students' knowledge readiness and limitations in using the concept.

One of the learning obstacles identified in this study was an ontogenic obstacle, particularly in the instrumental aspect. This obstacle was observed in the indicator of representing concepts in various forms of mathematical representation, where students experienced limitations in transforming information on the Cartesian coordinate plane into symbolic forms as the basis for determining the translated image. This difficulty indicates that students had not yet acquired sufficient prerequisite knowledge to support the appropriate construction of the concept of translation. From Suryadi's (2025) perspective, an ontogenic instrumental obstacle occurs when limitations in students' abilities or prior knowledge hinder their engagement in particular learning situations. Therefore, the obstacle identified in this study is not merely related to errors in determining coordinates but reflects limitations in the knowledge instruments required to establish connections between visual and symbolic representations in the concept of translation. This finding is consistent with the studies by Listiawan and Antoni (2021) and Setyani (2022), which stated that insufficient mastery of prerequisite knowledge is one of the factors contributing to learning obstacles in geometric transformation topics.

In addition to obstacles related to limitations in prerequisite knowledge, this study also identified epistemological obstacles in students' understanding of the concept of translation. These obstacles were observed in the indicators of restating concepts, applying particular procedures, and applying concepts in problem-solving. Based on the analysis, students had acquired initial knowledge of translation, such as knowing the definition and basic solution procedures; however, this knowledge could not yet be used flexibly when the concept was presented through different representations or situations.

According to Suryadi (2025), epistemological obstacles emerge when knowledge possessed by students can only be used in particular contexts but becomes limited when applied to situations with different demands. This characteristic was observed when students were able to state the definition of translation but were unable to explain the meaning of the concept based on visual representations. The same obstacle was also evident when students applied composition of translations procedures without understanding the relationships among transformations and when they were unable to apply the concept of translation to construct mathematical representations in contextual problems. Therefore, the epistemological obstacles identified in this study indicate that students' understanding of translation remained limited to certain forms of knowledge and had not yet developed into conceptual understanding that could be applied flexibly across various situations. This finding is consistent with the studies by Manandhar et al. (2022) and Mutawah et al. (2019), which showed that epistemological obstacles in geometric transformation topics emerge when students only master solution procedures for routine problems but experience difficulties when concepts are presented through different representations or contexts.

Based on the identification of learning obstacles, students' difficulties in understanding the concept of translation were not only related to their inability to perform transformation procedures but also to the processes of constructing and using knowledge progressively. The identified ontogenic obstacle indicates that limited mastery of prerequisite knowledge, particularly in understanding coordinate representations, became an initial barrier in constructing the concept of translation. Meanwhile, the epistemological obstacles indicate that students' existing knowledge of translation remained limited to particular contexts and could not yet be used flexibly when the concept was presented through different representations, procedures, or problem-solving situations.

Based on the identified ontogenic obstacle, the didactical design needs to consider students' prior knowledge readiness by providing activities that establish connections between prerequisite concepts and new concepts. In the context of translation, learning activities should facilitate students' understanding of the relationship between the position of objects on the coordinate plane and symbolic representations in the form of coordinates and translation vectors. Meanwhile, the epistemological obstacles highlight the importance of didactical designs that provide opportunities for students to develop conceptual understanding through variations in representations and situations. Therefore, translation learning should be designed not only to emphasize procedural mastery but also to include activities that enable students to construct conceptual meanings, connect various representations, and apply concepts in different contexts.

CONCLUSION

This study shows that students' conceptual understanding of translation had not yet been achieved optimally. Students' difficulties were identified in restating concepts, representing concepts in various forms of mathematical representation, applying translation procedures, and applying concepts in contextual problem-solving. The analysis of learning obstacles revealed that the identified obstacles consisted of ontogenic instrumental obstacles and epistemological obstacles. The ontogenic instrumental obstacle was associated with students' limited mastery of prerequisite knowledge required to represent the concept of translation, whereas the epistemological obstacles indicated students' limitations in using knowledge of translation flexibly across various representations and contexts. Based on the scope of affected indicators, epistemological obstacles emerged as the more dominant type of obstacle because they were identified in students' abilities to restate concepts, select procedures, and apply concepts in problem-solving.

This study represents the initial stage of the Didactical Design Research (DDR) framework, namely the identification of learning obstacles, which serves as an empirical foundation for designing a didactical design. The findings provide information about the characteristics of students' learning obstacles in translation that can serve as considerations for teachers and researchers in developing learning processes that are more responsive to students' learning needs. However, this study did not include the development or implementation stages of a didactical design; therefore, the design principles obtained remain as implications derived from the identified learning obstacles. Further research should develop, implement, and evaluate a didactical design based on the identified learning obstacles to examine its potential to minimize learning obstacles and support the development of students' conceptual understanding.

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